

Cubic B-splines for Isogeometric Analysis of Laminated Composite Plates Subjected to Hygroscopic Loading Using Classical Laminated Plate Theory

KNV Chandrasekhar^{1*}, V. Bhikshma², Madhusudhana Chary³, Maduri Venkat Sai⁴

Abstract

Composite laminates are inevitably used in several engineering structures due to their high strength and stiffness to weight ratio. The laminates are subjected to change in moisture content and the flexural behavior leads to change in the strength. Presence of moisture may induce residual stresses and strains. Isogeometric analysis is a well-proven method to perform analysis of laminated composite plates. The main focus of this study is to apply isogeometric analysis to determine the flexural response of a simply supported laminated plate having different lamina subjected to a change in moisture content of 1.5%. Classical laminated plate theory without including shear deformation is applied to thin plates in this study. Cubic b-splines were used to model the geometry of the plate. Four different cases of lamina, two symmetric cases which are 0/90/90/0 and 45/-45/-45/45 and two anti-symmetric cases of lamina which are 0/90/0/90 and 45/-45/45/-45 have been considered in this study. The bending characteristics of the thin plates which are deformed profile of the laminate along with the distribution of strain energy are shown. The maximum and minimum bending moment are calculated. The results obtained using isogeometric analysis are in good agreement with those results given in the literature obtained using finite element analysis performed with six node linear strain triangular elements.

Keywords: Hygroscopic, Cubic b-splines, Laminates, Simply supported, Symmetric, Anti-symmetric

INTRODUCTION

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The use of laminates is inevitable for use with engineering structures. Laminates are used in ships, aircrafts, missiles, automobiles, mining equipment, railway wagons, civil structures. The advantages of light weight, high strength, stability, corrosion, resistance, ease of fabrication [1], and desired mechanical properties have increased the use of laminates at an affordable cost. The use of cutouts allow the engineer to inspect the underlying surface and allow access ports for mechanical systems [2]. Laminates are used for passage of liquids in liquid retaining structures [3]. Laminates are glued at the soffit of the bridge decks [4]. During the service life, the laminates are subjected to different environmental conditions such as change in moisture content and temperature which will have an adverse effect on the performance of the laminate. Strength and stiffness are effected adversely with the change in the moisture content. The analysis of laminated

plates subjected to change in moisture content has been the subject of research in the recent past few years.

Isogeometric analysis (IGA) is introduced by Hughes [5], a numerical method useful to solve partial differential equations. This method is more advantages than other method as the domain can be represented precisely using Splines. The words “Iso” means same and “geometric” means the basis functions used to represent the geometry can be used to interpolate the field variables with in the domain, such as displacement at any point within the domain. Initial design domain can be represented using CAD and the structural analysis can be performed without any need of meshing.

The main purpose of this research is to apply isogeometric analysis and perform the structural analysis of laminated composite plates subjected to changes in moisture content. The response of the structure is measured in terms of displacement, strain energy, and bending stresses. Section 1 gives a brief introduction to this topic. Section 2 covered the literature review on hygroscopic loading of laminates. Section 3 presents the necessary theoretical background formulation required to form the stiffness matrix and the hygroscopic force vector. Section 4 presents the methodology followed to conduct this study. Section 5 presents the problem statement. Analysis is performed in section 6 and the results are presented. The deformed profiles of the laminates are presented. Section 7 concludes this study on using cubic b-splines to perform the analysis of SSSS laminated composites subjected to hygroscopic loading. The complete list of references are given at the end.

Objectives of the Study

- A. To determine the deflections for laminated plates subjected to hygroscopic loading using splines
- B. To study the Strain Energy distribution for plates with 3D images of deformed shape
- C. To study the variation of Bending moments (M_x , M_y , M_{xy}) at a chosen section

Scope of the Study

- The study is limited to linear static analysis only
- The buckling of shells is not considered in this study
- The material is free from any imperfections
- The temperature remains constant

LITERATURE REVIEW

In this paper, Sriram [6] performed the finite element analysis using eight noded quadrilateral isoparametric plate bending element to study the effect of moisture and temperature on the bending behavior of laminated composite plates. The conventional finite element formulation includes hygrothermal strains and transverse shear deformation in the analysis. The results are presented for anti-symmetric cross ply and angle ply laminates consisting of graphite/epoxy laminate subjected to uniform distribution of moisture and temperature. Shamshuddin et al. [7] in their research used exponential shear deformation theory to perform thermal stress analysis of laminated cross ply composite plates. Pranati [8] performed a numerical analysis which incorporates the shear deformation as well as transverse normal shear strains to conduct hygrothermal analysis of crossply laminates subjected to linear or gradient thermal profile across the thickness. Naidu and Sinha [9] present finite element formulation for a doubly curved shells and includes large deformations with Green Lagrange strains with six degrees of freedom at each node. The ratio of central deflection to thickness increases with increase in temperature. Kant and Kommineni [10] applied higher order shear deformation theory to analyse the fibre reinforced composite and sandwich laminates. This theory considered parabolic distribution of transverse shear strain through the thickness of the laminate. Marquest and Creus [11] employed a degenerated finite element model to analyse plates and shells. The material is considered as linear viscoelastic. Analysis is performed for both large displacement and small displacement formulations. Rath [12] did his research on parametric

resonance behavior of laminated composite curved panels subjected to adverse hygrothermal environment. Shubhankar and Anup [13] performed the mechanical and hygrothermal analysis to determine the transverse displacements, normal stresses and moments for four layered cross ply or angle ply laminated composite square plate when the edges are simply supported or clamped. Choudhury and Mondal [14] did their research on failure analysis to determine the first ply failure load at which the first ply fails. Njuguna, Banerjee [15] performed modal analysis of laminated beams using dynamic stiffness and finite element methods. The experiments have been carried out using impulse hammer techniques. Arash and Asghar [16] presented their research on the interlaminar normal and shear stresses which play a dominant role in the failure of composite laminates. Rath [17] presented the mathematical formulation for buckling effects of laminated composite plates subjected to moisture and temperature. Whitney and Ashton [18] presented their research on the effect of inducing residual stress, expansional strains on bending, buckling and vibrational analysis of high performance composite laminates. The authors [19] have performed their research on the non-linear dynamic response of laminated shells with imperfections. Singh and Chakrabarti [20] in their study on simply supported cross ply (0/90/0) square laminates subjected to sinusoidal temperature distribution in x-y plane and linearly varying through thickness have observed that with the decrease in the thickness ratio, the displacements and stresses have increased. Ashrof [21] studied the response of multilayered angle ply composite plates due to variation in temperature and moisture concentrations. Rajesh and Dharamveer [22] did their research study on buckling of composite laminated panels for economy, safety and reliability of the structures and components in their operational life especially for sensitive aerospace engineering applications.

THEORETICAL BACKGROUND

In the previous section, the literature is reviewed. In this section, the formulation of laminated composite plates for classical plate theory with no shear contribution is also presented.

Let Ω be the domain in R^2 by the mid plane of the plate and u, v, w denote the displacement components in x, y, z directions respectively. Using the Kirchhoff theory [23] the displacements at any point can be expressed as

$$\begin{aligned} u(x, y, z) &= u_o(x, y) - z\theta_x(x, y) \\ v(x, y, z) &= v_o(x, y) - z\theta_y(x, y) \\ w(x, y, z) &= w(x, y) \end{aligned} \quad (1)$$

where, $\theta_x = \frac{\partial w}{\partial x}$ and $\theta_y = \frac{\partial w}{\partial y}$

In-plane strains through the following equation

$$\varepsilon = [\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}]^T = \varepsilon_o + z\kappa$$

where, ε_o, κ are the in-plane deformations and curvatures of the middle surface respectively

$$\varepsilon_o = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} u_o \\ v_o \\ w \end{bmatrix} \quad \text{and} \quad \kappa = \begin{bmatrix} 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & 0 & -\frac{\partial^2}{\partial y^2} \\ 0 & 0 & -2\frac{\partial^2}{\partial x \partial y} \end{bmatrix} \begin{bmatrix} u_o \\ v_o \\ w \end{bmatrix} \quad (2)$$

$$\bar{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta$$

$$\begin{aligned}
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\
\bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta)
\end{aligned}$$

with

$$\begin{aligned}
Q_{11} &= \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, & Q_{12} &= \frac{\nu_{12}E_{11}}{1 - \nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, & Q_{44} &= G_{13} \\
Q_{55} &= G_{23}, & Q_{66} &= G_{12}
\end{aligned} \tag{3}$$

$$\nu_{21}E_{11} = \nu_{12}E_{22}$$

E_{11} and E_{22} are Young's modulus of elasticity parallel and perpendicular to the fibre orientation. G_{12} is the shear modulus and are ν_{12} and ν_{21} are Poissons' ratios [24]. As shown in the Figure 1, the layers of lamina are given, the distance is measured from the centre of the lamina.

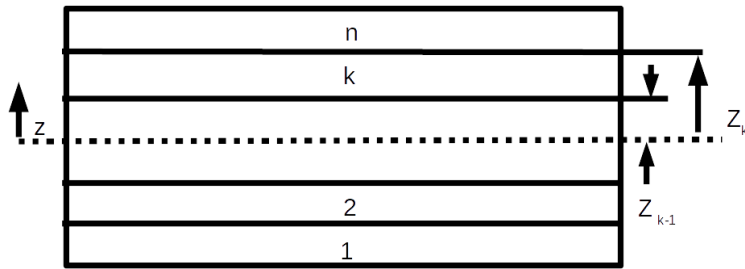


Figure 1. Showing the layers of lamina.

$$[E] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k - z_{k-1}) \quad B_{ij} = \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \quad D_{ij} = \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \tag{4}$$

The value of z is measured from the centre of the lamina as shown in the Figure 1.

Using the NURBS basis functions, both the description of the geometry for the physical and the displacement field are expressed as

$$x^h(\xi, \eta) = \sum_A^{nm} R_A(\xi, \eta) P_A \quad u^h(x(\xi, \eta)) = \sum_A^{nm} R_A(\xi, \eta) q_A \tag{5}$$

where, $n \times m$ is the number of basis functions.

$x^T = (x \ y)$ is the physical coordinate vector, $R_A(\xi, \eta)$ is rational basic functions and $q_A = [u_A \ v_A \ w_A]^T$ is the degrees of freedom of u^h associated to the control point A.

The strains can be expressed to following nodal displacements as

$$[\varepsilon_o \ \kappa]^T = \sum_A^{nm} [B_A^m \ B_A^b]^T q_A$$

$$B_A^m = \begin{bmatrix} R_{A,x} & 0 & 0 \\ 0 & R_{A,y} & 0 \\ R_{A,y} & R_{A,x} & 0 \end{bmatrix} \quad \text{and} \quad B_A^b = \begin{bmatrix} 0 & 0 & -R_{A,xx} \\ 0 & 0 & -R_{A,yy} \\ 0 & 0 & -2R_{A,xy} \end{bmatrix} \quad (6)$$

B_A^m and B_A^b are membrane and bending strain displacement matrices using the derivatives of the basis function [24]

$$\text{Global stiffness matrix } K = \int_{\Omega} \left\{ \begin{bmatrix} B^m \\ B^b \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} B^m \\ B^b \end{bmatrix} \right\} d\Omega$$

Hygro Thermal Analysis

$$\text{Strain Tensor } \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2} \gamma_{12} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2} \gamma_{xy} \end{bmatrix} \quad (7)$$

$$[T] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \frac{1}{2} \beta_{xy} \end{bmatrix} = [T^{-1}] \begin{bmatrix} \beta_1 \\ \beta_2 \\ \frac{1}{2} \beta_{12} \end{bmatrix} = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & -2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & 2 \sin \theta \cos \theta \\ \sin \theta \cos \theta & -\sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \frac{1}{2} \beta_{12} \end{bmatrix}$$

where, $\frac{1}{2} \alpha_{12} = \frac{1}{2} \beta_{12} = 0$

$$\text{Hygro strain} = \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} = \begin{bmatrix} \Delta C \beta_x \\ \Delta C \beta_y \\ \Delta C \beta_{xy} \end{bmatrix}$$

Total strain = Mechanical strain + thermal Strain + Hygro strain

$$\begin{bmatrix} \varepsilon_x^{Tot} \\ \varepsilon_y^{Tot} \\ \gamma_{xy}^{Tot} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^{Mech} \\ \varepsilon_y^{Mech} \\ \gamma_{xy}^{Mech} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Th} \\ \varepsilon_y^{Th} \\ \gamma_{xy}^{Th} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} \quad (9)$$

The total strain is given as

$$\begin{bmatrix} \varepsilon_x^{Tot} \\ \varepsilon_y^{Tot} \\ \gamma_{xy}^{Tot} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Th} \\ \varepsilon_y^{Th} \\ \gamma_{xy}^{Th} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} \quad (10)$$

The stress is given as

$$\begin{aligned} [\sigma] &= [\bar{Q}][\varepsilon^{Tot}] = [\bar{Q}](\{\varepsilon^0\} + z\{k\} - \{\varepsilon^{Th}\} - \{\varepsilon^{Hy}\}) \\ \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} &= \begin{bmatrix} N^{Th} \\ M^{Th} \end{bmatrix} + \begin{bmatrix} N^{Hy} \\ M^{Hy} \end{bmatrix} = [F^N] \\ (A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} (1, z, z^2) \bar{Q}_{ij} dz \quad i, j = 1, 2, 6 \end{aligned} \quad (11)$$

The element level nodal load vector due to hygroscopic forces is given as $\iint [B]^T \{F^N\} dx dy$

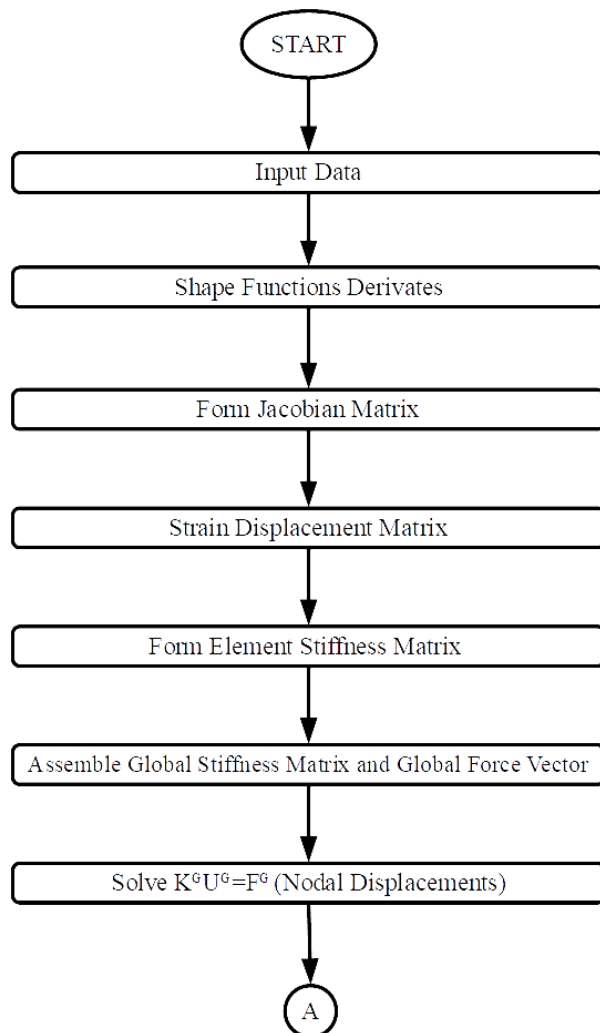


Figure 2. Flowchart showing the steps to determine the nodal displacement.

METHODOLOGY

Figure 2 shows the steps followed to perform static analysis of laminated composite plates. The element stiffness is formulated first and then assembled to form global stiffness matrix. Similarly the

force vector for each element is assembled to form a global force vector. The nodal displacements can be determined and the deformed profile can be obtained. The finite element analysis is performed using six node linear strain triangular elements. The isogeometric analysis is performed using cubic b-splines. The results obtained are then compared.

PROBLEM STATEMENT

1. To analyse SSSS plate having different lamina 0/90/0/90, 0/90/90/0, 45/-45/45/-45, 45/-45/-45/45 subjected to change in moisture content 1.5%
2. To study the bending characteristics namely Displacement, Bending Moment, and Strain Energy

Notations

The support notations for the simply supported support is as shown in Figure 3.

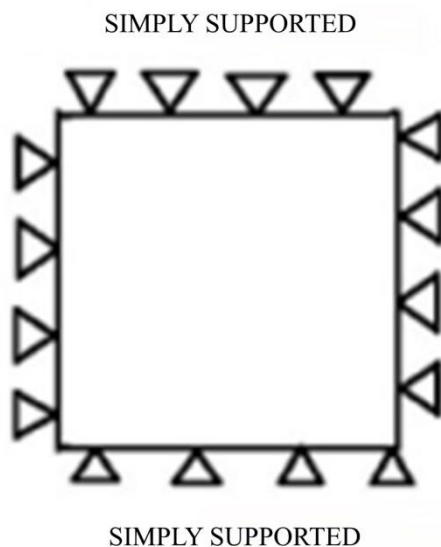


Figure 3. Notation for simply supported plate along the edges.

ANALYSIS OF SSSS PLATE SUBJECTED TO 1.5% CHANGE IN MOISTURE CONTENT

The analysis of the SSSS laminated plate subjected to hygroscopic loading is performed here.

Input Data

- The input data for the SSSS laminated plate is given here
- The ratio of sides (a/b) ratio is equal to 1
- The ratio of side to the thickness (a/t) ratio is equal to 100
- The Young's modulus of elasticity in direction 1 = $E_{11} = E_{xx} = 130000$ MPa
- The Young's modulus of elasticity in direction 2 = $E_{22} = E_{yy} = 8500$ MPa
- The Shear Modulus of elasticity in direction 12 = $G_{12} = G_{xy} = 6000$ MPa
- The Shear Modulus of elasticity in direction 13 = $G_{13} = G_{xz} = 6000$ MPa
- The Shear Modulus of elasticity in direction 23 = $G_{23} = G_{yz} = 3000$ MPa
- The Poisson's ratio ν_{12} is equal to 0.3
- The number of divisions along x-axis (direction 1 & direction 2) is equal to 10
- The mesh is made of 6 node linear strain triangular elements for finite element analysis
- The value of $\beta_1 = 0$ and $\beta_2 = 0.44$
- The change in moisture content is equal to $\Delta C = 1.5\% = 0.015$
- The elastic moduli of graphite epoxy lamina at different moisture concentrations are given in Table 1.

Table 1. Elastic moduli of graphite/epoxy lamina at different moisture concentrations [3]

$$G_{13} = G_{12}, G_{23} = 0.5G_{12}, \nu_{12} = 0.3, \beta_1 = 0, \beta_2 = 0.44$$

Elastic Modulus	Moisture Concentration						
	0.0%	0.25%	0.5%	0.75%	1.0%	1.25%	1.5%
E_1 (GPa)	130	130	130	130	130	130	130
E_2 (GPa)	9.5	9.25	9.0	8.75	8.5	8.5	8.5
G_1 (GPa)	6	6	6	6	6	6	6

Hygroscopic Analysis

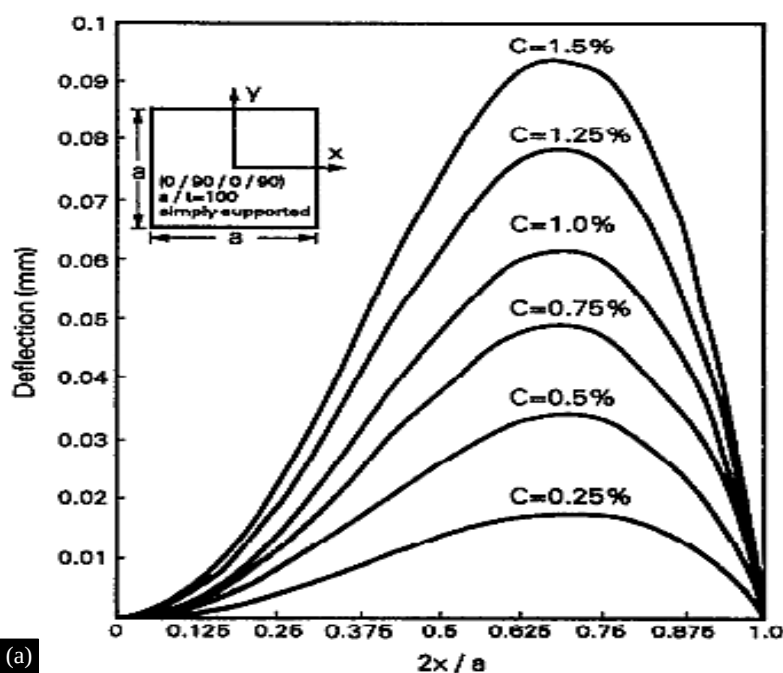
The analysis of SSSS laminated plate subjected to 1.5% change in moisture content is performed here.

SSSS laminated plate having 0/90/0/90 lamina

The problem statement is taken from Sairam [6] and solved here for the case of SSSS laminated composite plate subjected to 1.5% change in moisture content. Figure 4a shows the deformed profile given by Sairam [6]. Figure 4b shows the distribution of strain energy in the plate. Figure 4c shows the deformed profile of the SSSS plate subjected to 1.5% hygroscopic loading and analysed using classical plate theory. The nodal stress calculated using the centroidal stresses at each element are lower than the stresses calculated at the corners using the extrapolation method discussed in the literature. The Moments M_x , M_y at the centre of the plate are equal and opposite. The Moment M_x and M_y at the centre of the plate are equal in magnitude and opposite in sign. The magnitude of moment is equal to 7.3635 N-mm. The moment resultant M_{xy} at the centre of the plate is equal to zero. The maximum displacements using isogeometric analysis are found to be in close agreement with the displacements obtained using finite element analysis as shown in Table 2.

Table 2. Maximum deflection for SSSS laminated plate having 0/90/0/90 lamina

Method used to perform analysis	Max. deflection (mm)	Ratio IGA/FEA
FEA using six node LST 20×20 elements [23]	0.09215 [23]	
IGA using cubic splines and CLPT 24×24 elements	0.095	1.030



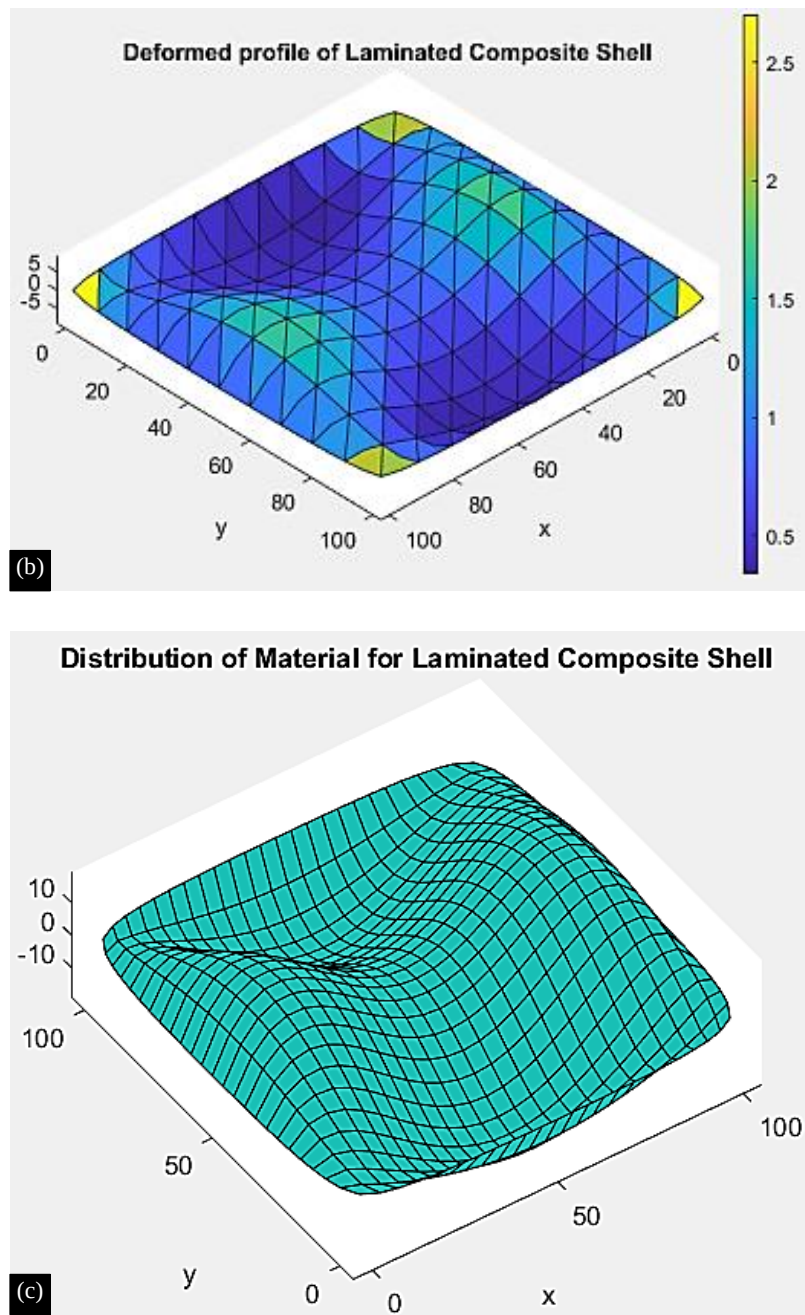


Figure 4. The deformed shape of SSSS square laminated plate having 0/90/0/90 lamina subjected to hygroscopic loading.

(a) FEA using Quad 8 [3] (b) distribution of Strain energy using FEA using 6N LST [23] (c) Cubic b-splines and CLPT

SSSS laminated plate having 0/90/90/0 lamina

The deformed profile of SSSS laminated composite plate subjected to hygroscopic loading of 1.5% is as shown in the Figure 5a. The plate is analysed using finite element analysis and Isogeometric analysis. The six node linear strain triangular elements are used to perform the finite element analysis. Cubic b-splines are used to perform the Isogeometric analysis. Figure 5a shows the deformed shape of the plate using finite element analysis and Figure 5b shows the deformed shape using cubic b-splines. There is no displacement in the vertical direction. The deformed shape clearly shows the similarity in the results obtained using finite element analysis and isogeometric analysis using cubic b-splines.

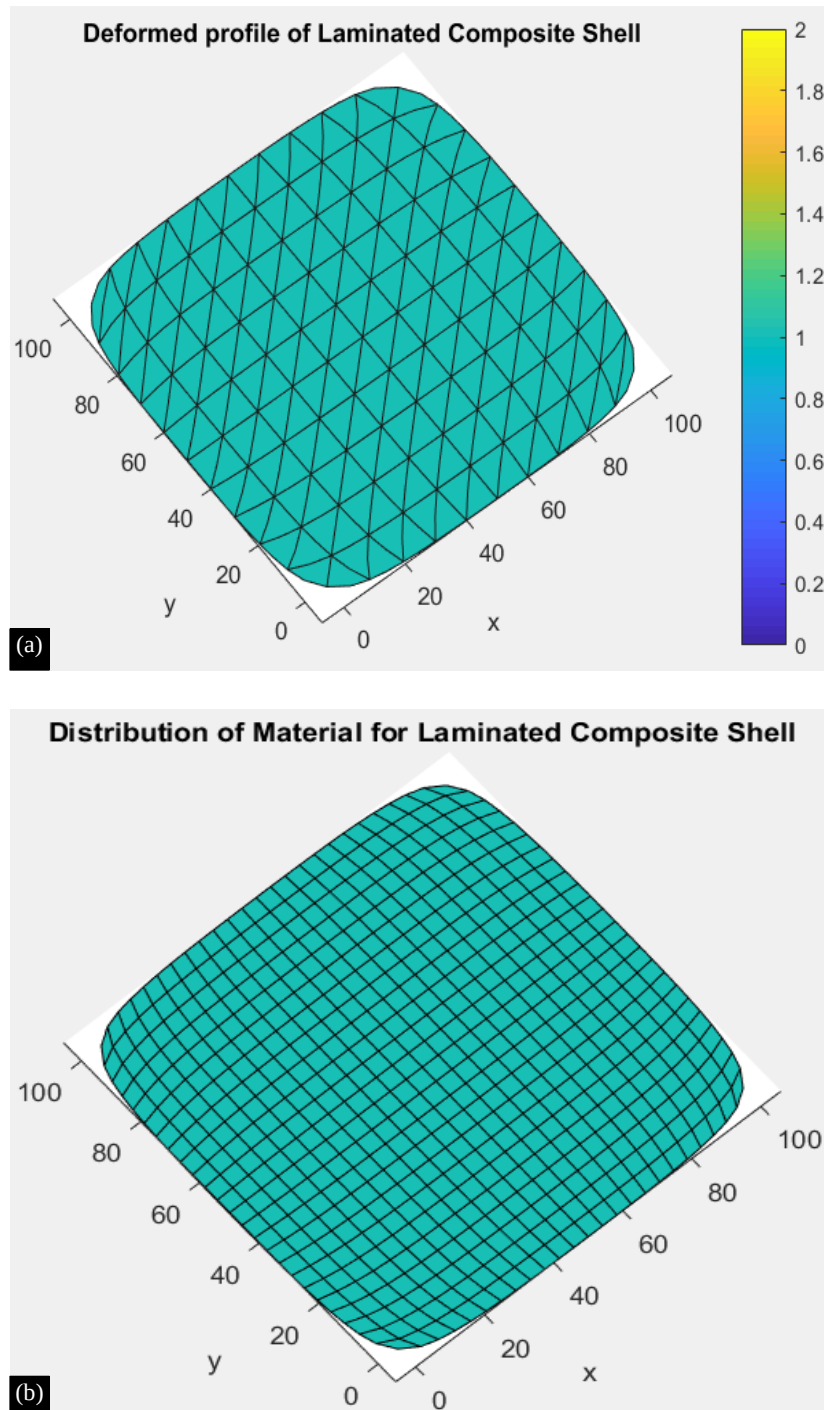


Figure 5. deformed shape of SSSS square laminated plate having 0/90/90/0 lamina.
(a) Using 6N LST (b) Using Cubic b-splines with CLPT

SSSS laminated plate having 45/-45/45/-45 lamina

The deformed shape of the plate is as shown in the Figures 6a and 6b. The deformed shape clearly shows a similarity between the results obtained using finite element analysis and Isogeometric analysis. The maximum deflection using six node linear strain triangle elements is found to be 0.04234. The maximum deflection using cubic b-splines is found to be 0.04381 as shown in Table 3. The crest pattern is observed along the diagonal and similarly a trough pattern is observed along the other diagonal as shown in Figure 6.

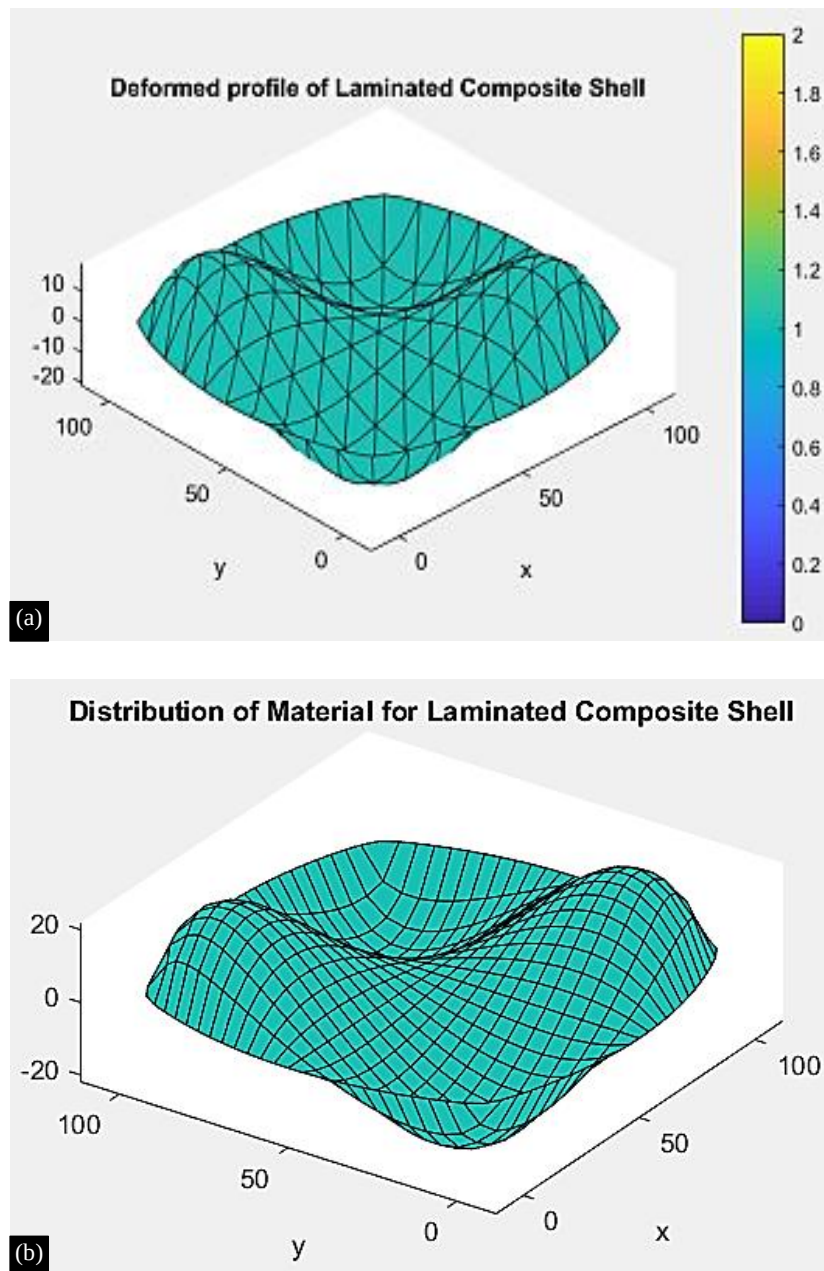


Figure 6. Deformed shape of SSSS square laminated plate having 45/-45/45/-45 lamina.
(a) Using 6N LST (b) Using cubic b-splines and CLPT

Table 3. showing the maximum vertical deflection using 6N LST and cubic splines

Method of analysis	FEA using 6N LST	IGA using Cubic b-splines	Ratio IGA/FEA
Max. vertical deflection	0.04234	0.04381	1.034

SSSS laminated plate having 45/-45/-45/45 lamina

The deformed shape using finite element analysis is as shown in the Figure 7a and using isogeometric analysis with cubic b-splines as shown in Figure 7b. The similarity of the results clearly show that the plate stretch along one diagonal and compresses along the other diagonal. The displacement in the vertical direction is equal to zero and only in plane displacements are being observed.

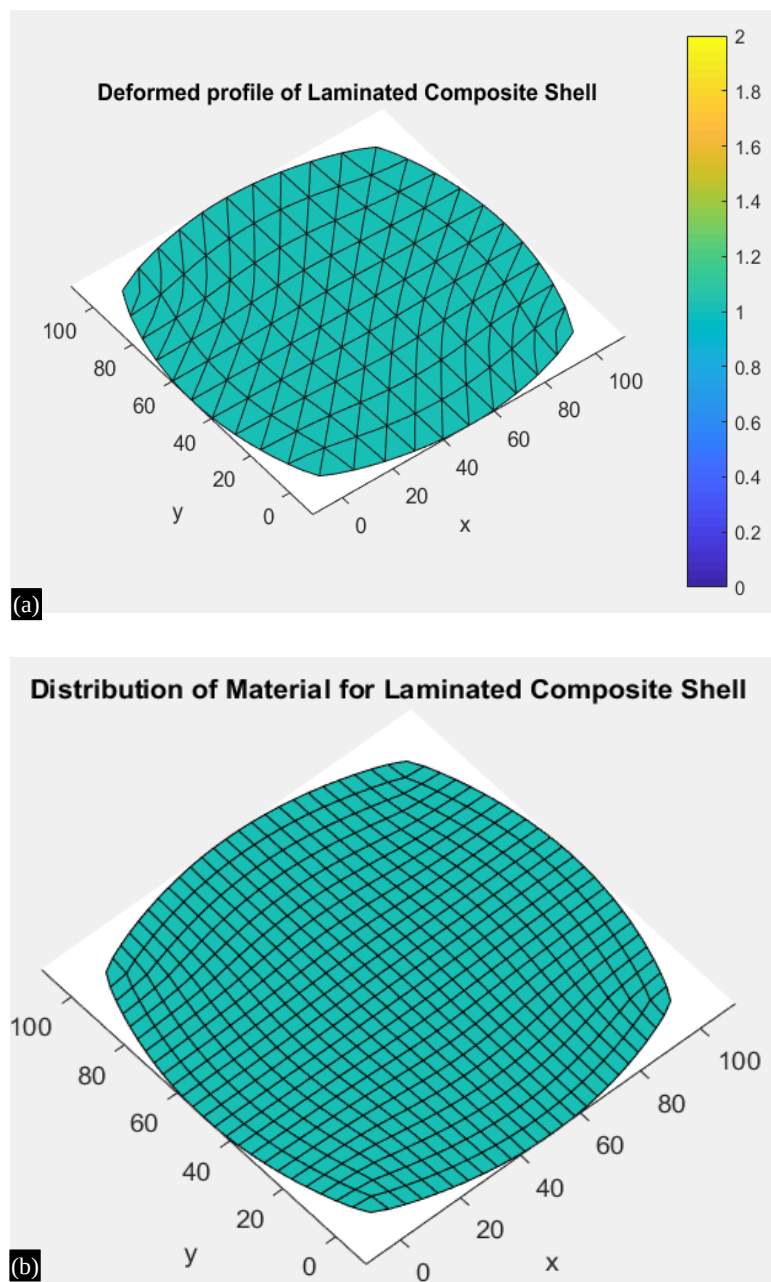


Figure 7. Deformed shape of SSSS square laminated plate having 45/-45/-45/45 lamina.
(a) Using 6N LST (b) Using cubic b-splines and CLPT

CONCLUSIONS

SSSS laminated composite plate is considered in this study. The plate is analysed using finite element analysis and isogeometric analysis. A square plate is considered with the side to thickness ratio is taken as 100. To perform finite element analysis, the domain is discretised using six node triangular elements. To perform isogeometric analysis, cubic b-splines are used. The plate is subjected to hygroscopic loading of change in moisture content as 1.5%. The analysis is performed using four different type of lamina namely 0/90/0/90, 0/90/90/0, 45/-45/45/-45, 45/-45/-45/45. The deformed profile and the maximum displacements are compared.

In case of SSSS plate having 0/90/0/90 lamina, the results of deflection in the deformed profile are compared to those given in the literature by Sairam using eight node quadrilateral elements. The

results were in close agreement and the results of finite element analysis using six node triangular elements and isogeometric analysis using cubic b-splines are in close agreement. In case of SSSS laminated plate having 45/-45/45/-45 the deformed profile clearly shows a wavy pattern, there is a hump along one diagonal and there is trough along the other diagonal. The deformed profile obtained using finite element analysis is clearly in close agreement with the deformed profile obtained using isogeometric analysis.

In case of symmetric lamina, for 0/90/90/0 and 45/-45/-45/45 there is no vertical displacement the deformed profile of the lamina show that there is only planar displacements in x and y plane. The deformed profile obtained using finite element analysis is in close agreement with the deformed profile obtained using isogeometric analysis using cubic b-splines.

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