

Thermal Analysis of Isotropic and Orthotropic Plate by Trigonometric Shear Deformation Theory

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Abstract

In the present study, a sinusoidal shear and normal deformation theory taking into account effects of transverse shear as well as transverse normal is used to develop the analytical solution for the bidirectional bending analysis of isotropic, transversely isotropic, laminated composite and sandwich rectangular plates. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The Navier solution for simply supported laminated composite plates have to develop. Results obtained for displacements and stresses of simply supported rectangular plates have to compare with those of other refined theories and exact elasticity solution wherever applicable. The Navier-type exact solutions for static bending analysis are have to for sinusoidally and uniformly distributed loads. The accuracy of the present theory is ascertained by comparing it with various available results in the literature.

Keywords: Trigonometric shear deformation theory, transverse in-plane stress, transverse displacement, trigonometric function

INTRODUCTION

Composite materials are obtained by a combination of two or more materials which together have characteristics that separately usually cannot be achieved with reasonable costs. During the last decades, a sudden increase of the use of modern composite materials is based on the achieved advanced properties which include their lower weight and higher strength and resistance compared to the classical and conventional composite materials. It is clear that the modern composite materials are materials of the future, because they allow the designers to choose the appropriate characteristics of the elements structure depending on the applicable problem. Due to its complexity, the researches of modern materials involve several scientific fields, and each of them, in their own way, contributes to their development. Laminated composite and sandwich structures are extensively used in many fields such as aerospace, naval, automotive, and civil industries, for their superior performance and reliability. In last decades the use of sandwich construction grew rapidly around the world. Its

advantages, the development of new materials and the necessity of high performance under static, dynamic and thermal loads guarantee that the sandwich structures will be in demand for many years. With the increased use of sandwich structures, there is a tremendous need to develop efficient manufacturing techniques, economical and effective repair techniques, and analysis methods to predict the short and long-term behaviour of the multilayer composite materials under a variety of loading and environmental conditions. The use of composite material for the design of structures in aerospace, civil, mechanical, marine and automotive applications has improved the performance and reliability of

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structural system due to their mechanical advantage of specific modulus and specific strength over monolithic materials, improved fatigue and impact resistance, and design flexibility. Laminated composite, one of the types of composite materials, consist of layers of at least two different materials that are bonded together. Laminated composite plates, a class of laminated fibrous composites, are made of laminate, each laminate consisting of parallel fibres embedded in a matrix material [1–8].

REVIEW OF LITERATURE

Composite laminated structures are widely used, particularly in aerospace engineering due to their superior mechanical properties. The components of such structures are often subjected to combinations of transverse mechanical and thermal loads. To address the correct response of composite laminated plates when subjected to combined loads is of prime interest to structural analysis. By virtue of their high strength to weight ratios and because of their mechanical properties in various directions, they can be tailored as per requirements. Further, they combine a number of unique properties, including corrosion resistance, high damping, temperature resistance and low thermal coefficient of expansion. These unique properties have resulted in the expanded use of the advanced composite materials in structures subjected to severe thermal environments. These structures are usually referred to as high-temperature structures. Examples are provided by structures used in high-speed aircraft, spacecraft, etc. The high velocities of such structures give rise to aerodynamic heating, which produces intense thermal stresses that reduces the strength of the aircraft structure. The composite materials provide excellent thermo-mechanical properties at elevated temperatures. Coefficients of thermal expansion in the direction of fibres are usually much smaller than those in the transverse direction. This results in high stresses at the interfaces. In order to describe the correct thermo-mechanical response of laminated plates including shear deformation effects, refined theories are required. The objective of this paper is to present an equivalent single-layer shear deformation theory for evaluation of displacements and stresses of cross-ply laminated plates subjected to uniformly distribute non-linear thermo-mechanical load. A trigonometric shear deformation theory is used. The in-plane displacement field uses a sinusoidal function in terms of the thickness coordinate to include the shear deformation effect. The theory satisfies the shear stress-free boundary conditions on the top and bottom surfaces of the plate. The present theory obviates the need of a shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. Stresses and displacements for orthotropic, two-layer ant symmetric, and three-layer symmetric square cross-ply laminated plates subjected to uniformly distribute non-linear thermo-mechanical load are obtained. Numerical results of the present theory for displacement and thermal stresses are compared with those of classical, first-order and higher-order shear deformation plate theories [9].

In the literature, different models were proposed in order to study multi-layered composite structures such as equivalent single-layer, quasi-layer wise and layer wise models. The heterogeneous laminated plates and shells are treated as a statically equivalent single-layer, thus reducing the 3D problem to a 2D one. Another important point in the analysis of composite structures is the variational statement used in the analysis to derive the necessary governing equations and the boundary conditions. Deformation theories can either be developed using displacement-based theories (when the principle of virtual displacement is used), stress-based theories or displacement-stress-based theories. The well-described unified formulation, initially presented by Carrera and extended by Demasi describes precisely and clearly the models, types, and class of theories. Recently, using Carrera unified formulation (CUF) and an asymptotic expansion method have discussed the effectiveness of higher-order terms in refined beam theories. In a similar fashion, CUF have found the so-called best plate theory, but this time considering an axiomatic hypothesis method [10].

According to the generalized unified formulation (GUF) proposed among equivalent single-layer theories, there are many classes of theories. However, there are mainly three well-known major theories; namely the classical lamination theory (CLT), which is based on the assumptions of

Kirchhoff's plate theory which neglects the inter-laminar shear deformation, the first-order shear deformation theory (FSDT) assumes constant transverse shear deformation through the entire thickness of the laminate and violates stress-free boundary conditions at the top and bottom surfaces of the plate, and more accurate theories such as higher-order theories (HSDT) assume quadratic, cubic or higher (also non-polynomial, e.g. trigonometric) variations of surface-parallel displacements through the entire thickness of the laminates to model the behaviour of the structure for thick to thin regions.

In fact, few higher-order shear deformation theories containing non-polynomial shape strain functions were developed. In the present work, a new trigonometric higher-order shear deformation theory in which the displacement of the middle surface expanded as tangential trigonometric functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness is proposed. Necessary equilibrium equations and boundary conditions are derived by employing the principle of virtual work. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and the tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. Exact solutions for deflections and stresses of simply supported sandwich plates are presented by using Navier-type solution technique [11].

The classical laminated plate theory (CLPT) is based on the assumption that straight lines which are normal to the neutral surface before deformation remain straight and normal to the neutral surface after deformation. Since the transverse shear deformation is neglected, it cannot be suitable for the modeling of moderately thick or thick plates in which transverse shear deformation effects are more significant. To overcome the limitations of CLPT and accurately incorporate the transverse shear effects, many shear deformation theories have been developed. First-order shear deformation theories (FSDTs) are based on the assumption that straight lines which are normal to neutral surface before deformation remain straight but not necessarily normal to the deformed neutral surface. In these theories, the transverse shear strain distribution is assumed to be constant through the plate thickness and therefore it requires shear correction factor in order to satisfy traction free boundary conditions at top and bottom surfaces of plates. Ferreira have studied static deformations and free vibrations of shear flexible isotropic and laminated composite plates with a first order shear deformation theory. Civalek has developed the discrete singular convolution method for static analysis of thick symmetric cross-ply laminated composite plates based on the first-order shear deformation theory. Fares and Elmaghany presented a refined zigzag non-linear FSDT of composite laminated plates using a modified mixed variational formulation. The limitations of CLPT and FSDTs are stimulated the development of higher-order shear deformation theories to avoid the use of shear correction factors, to include effect of cross-sectional warping and to get the realistic variation of the transverse shear strains and stresses through the thickness of plates. An extensive review of laminated plate theories can be found has developed well-known higher-order shear deformation theory considering polynomial functions in-terms of thickness coordinate. Soldatos proposed a hyperbolic shear deformation theory for homogenous monoclinic plates whereas Meiche et al. proposed a new hyperbolic shear deformation theory for buckling and vibration analysis of functionally graded sandwich plates. A new inverse hyperbolic shear deformation theory is proposed by Grover et al. Daouadji et al. have developed hyperbolic shear deformation theory with four variables for the bending analysis of functionally graded plates. Karama et al. proposed an exponential function in terms of thickness coordinate for laminated composite beam and plates. Sayyad and Ghugal also studied an exponential shear deformation theory for the bending, buckling and free vibration analysis of isotropic plates. a Navier-type solution for the exact three-dimensional solution for isotropic thick and thin rectangular plates. Levy type solution of cross-ply plates based on higher-order theory. The theories containing trigonometric (sinusoidal) functions involving thickness coordinate in the displacement fields of plate theories are designated as trigonometric or sinusoidal shear deformation theories. They have presented detail literature review on this class of shear deformation theory. In this paper, a sinusoidal shear and normal deformation theory (SSNDT) is used to develop the analytical

solution for the bidirectional bending analysis of transversely isotropic, laminated composite and sandwich plates. The present theory differs from other higher-order theories because, in present theory effect of transverse normal strain is included which is not assessed by the other researchers. Exact elasticity solutions for the bidirectional bending analysis of laminated composite plates are given by Pagano, Zenkour and Valek and Baltacioglu. The results of present theory are compared with those of other refined theories and exact solution given by Zenkour to assess the validity of the present theory [12].

A sinusoidal shear and normal deformation theory taking into account effects of transverse shear as well as transverse normal is used to develop the analytical solution for the bidirectional bending analysis of isotropic, transversely isotropic, laminated composite and sandwich rectangular plates. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and traction free boundary conditions on the plate boundary surface, thus a shear correction factor is not required. Results obtained for displacements and stresses of simply supported rectangular plates are compared with those of other refined theories and exact elasticity solution wherever applicable [13].

The components of such structures are often subjected to combinations of transverse mechanical and thermal loads. To address the correct response of composite laminated plates when subjected to combined loads is of prime interest to structural analysis. The results are compared with classical plate theory, first-order shear deformation theory and higher-order shear deformation theory. The results of the present theory are in good agreement with those of higher-order shear deformation theory under uniformly distributed non-linear thermo-mechanical load. This paper is to present an equivalent single-layer shear deformation theory for evaluation of displacements and stresses of cross-ply laminated plates subjected to uniformly distributed non-linear thermo-mechanical load. A trigonometric shear deformation theory is used. The in-plane displacement field uses a sinusoidal function in terms of the thickness coordinate to include the shear deformation effect [14].

Developed a novel simulation approach to model electron mobility in the inversion layer which encompasses all the important effects of arbitrary wafer and applied stress orientations, such as carrier re-population, band warping, and scattering, going beyond the separate treatments of band warping and inversion anisotropy that have been demonstrated. Our model predicts an important consequence of electron band warping in retaining the increase of stress gain at high stress levels in the presence of shear stress at strong inversion [15].

This is the result of neglecting transverse shear strains. However, non-negligible shear deformations occur in thick and moderately thick plates and the theory gives inaccurate results for laminated plates. So, it is obvious that transverse shear deformations have to be taken into account in the analysis. One of the well-known plate theories is the Mindlin model which is a first-order shear deformation theory (FSDT) and takes the displacement field as linear variations of midspan displacements. In this theory, the relation between the resultant shear forces and the shear strains is affected by the shear correction factors [16].

A few higher-order theories and the first-order theory developed by other investigators and already available in the literature are also considered for the evaluation. The equations of equilibrium are obtained using principle of minimum potential energy (PMPE). Solutions are obtained in closed form using Navier's technique by solving the boundary value problem. The comparison of the present results with the available elasticity solutions and the results computed independently using the first-order and the other higher-order theories available in the literature shows that this refined theory predicts the transverse displacement and the stresses more accurately than all other theories considered in this paper. After establishing the accuracy of present results for composite and sandwich plates, new results for the stretching bending coupling behaviour of antisymmetric sandwich

laminates using all the theories considered in this paper are presented which will serve as a benchmark for future investigations [17].

To prove the credibility of the present theory, we applied it to the bending analysis of beams. A simply supported isotropic, laminated composite and sandwich beams are analysed using Navier approach. The numerical results of non-dimensional displacements and stresses obtained by using the present theory are presented and compared with those of other refined theories available in the literature along with the elasticity solution. In this paper, the refined beam theory (RBT) is examined for the bending of simply supported isotropic, laminated composite and sandwich beams. The axial displacement field uses parabolic function in terms of thickness ordinate to include the effect of transverse shear deformation [18].

METHODS FOR THERMAL ANALYSIS OF ISOTROPIC AND ANISOTROPIC PLATE

Following Methods are used for analysis.

Introduction

In this chapter trigonometric shear deformation theory is presented for the thermal analysis of isotropic, orthotropic, two-layer anti-symmetric and three-layer symmetric cross-ply laminated plates subjected to linear and non-linear thermal and thermo-mechanical loads. The in-plane displacements field uses sinusoidal function in terms of thickness coordinate to include shear deformation effect. The theory satisfies the shear stress-free boundary conditions on the top and bottom of surfaces of the plate. The present theory obviates the need of shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The displacement and stresses for isotropic, orthotropic, two-layer anti-symmetric, and three-layer symmetric square laminated plates are obtained subjected to single sinusoidal and uniformly distributed thermo-mechanical load. The validity of present theory is verified by comparing the results with those of classical plate theory, first-order shear deformation theory and higher-order shear deformation theory of Reddy [19–25].

Theoretical Formulation of Plates using Trigonometric Shear Deformation Theory (TSDT).

The theoretical formulation of plates based on certain assumptions is presented. The

Variation ally correct forms of differential equations and boundary conditions, based on assumed displacement field will be obtained using the principle of virtual work [26, 27].

The Plate Under Consideration

The geometry and layer numbering of the laminated plate under consideration is shown in Figure 1. The plate is of constant thickness and is composed of thin anisotropic layers perfectly bonded together. The material of each layer is assumed to possess a plane of elastic symmetry parallel to the plate [28–36].

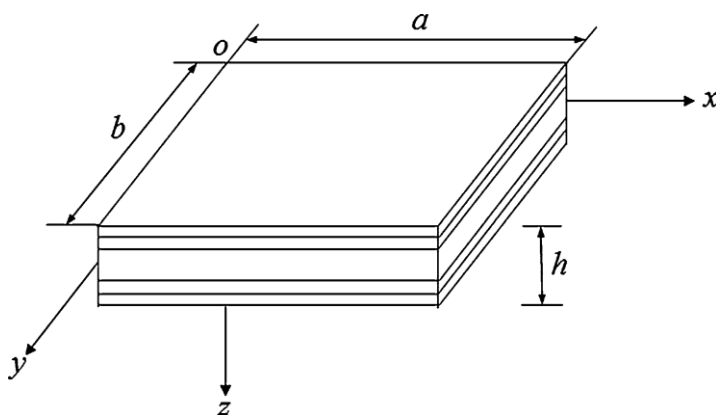


Figure 1. Plate geometry and coordinate system.

Consider a rectangular cross-ply laminated plate of length a , width b , and total thickness composed of orthotropic layers. The material of each layer is assumed to have one plan material property symmetry parallel to x - y plane. The coordinate system is such that the plane of the plate coincides with x - y plane, and z axis is normal to the middle plane. The r surface of the plate ($z = -h/2$) is subjected to a thermal load $T(x, y, z)$. The region of the in $(0-x, y, z)$ right-handed Cartesian coordinate system is

$$0 < x < a; 0 < y < b; -h/2 \leq z \leq h/2$$

Assumption of Trigonometric Shear Deformation Theory (TSDT)

1. The displacements (u -in x -direction, v in y -direction, w in z -direction) are small comparison with the plate thickness and, therefore, strains involved are infinitesimally results, normal strains result, normal strains ϵ_x, ϵ_y and shear strains $\gamma_{xy}, \gamma_{zy}, \gamma_{zx}$ can be expressed in terms of displacements u, v, w by using strain—displacement relations.
2. The transverse displacement w in z -direction is assumed to be a function of x, y .
3. The body forces are ignored in the analysis.
4. The plate is subjected to linear and non-linear thermal loads across the thickness of plate in combination with transverse mechanical load.
5. All layers are perfectly bonded with each other.
6. ϕ And ψ are rotations and are functions of x and y .

The Displacement Field

The displacement field of the present trigonometric shear deformation theory at a point located at (x, y, z) in the plate is of the form.

$$u(x, y, z) = u_0(x, y) - z \frac{\partial \omega_0(x, y)}{\partial x} + f(z)\phi(x, y) \quad (1)$$

$$v(x, y, z) = v_0(x, y) - z \frac{\partial \omega_0(x, y)}{\partial y} + f(z)\psi(x, y)$$

$$\omega(x, y, z) = \omega_0(x, y) \quad (2)$$

Where,

$$f(z) = \frac{h}{\pi} \sin \frac{\pi z}{h}$$

Here (u, v, w) are the axial displacements along x, y and z directions, and are functions of the spatial co-ordinates (u_0, v_0, ω_0) are the displacements of a point on the mid-plane, ϕ and ψ and are the rotations about the y and x axes in xz and yz planes due to bending. The generalized displacements ($u_0, v_0, \omega_0, \phi, \psi$) are functions of the (x, y) co-ordinates.

Strain-Displacement Relationship

The normal and shear strains are obtained within the framework of linear theory of elasticity. The infinitesimal strains associated with the displacement field (3.2) are as follows:

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 \omega_0}{\partial x^2} + f(z) \frac{\partial \phi_0}{\partial x}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 \omega_0}{\partial y^2} + f(z) \frac{\partial \psi_0}{\partial y}$$

$$\epsilon_{zz} = 0$$

Stress-Strain Relationship:

The stress-strain relationship for the k th layer lamina in a laminated plate under thermal loading can be written as

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix}_{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \epsilon_{xx} - \alpha_X T \\ \epsilon_{yy} - \alpha_X T \\ \vartheta_{xy} \end{Bmatrix}_{(k)}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_{(k)} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}_{(k)} \begin{Bmatrix} \vartheta_{yz} \\ \vartheta_{xz} \end{Bmatrix}_{(k)} \quad (3)$$

Normal Stress

$$\sigma_{xx} = Q_{11} \epsilon_{xx} + Q_{12} \epsilon_{yy}$$

$$\sigma_{yy} = Q_{21} \epsilon_{xx} + Q_{22} \epsilon_{yy}$$

$$\sigma_{zz} = Q_{13} \epsilon_{xx} + Q_{23} \epsilon_{yy}$$

Shear stress

$$\tau_{xy} = \tau_{yx} = Q_{66} \vartheta_{xy}$$

$$\tau_{xz} = \tau_{zx} = Q_{55} \vartheta_{xz}$$

$$\tau_{yz} = \tau_{zy} = Q_{44} \vartheta_{yz}$$

Where,

Q_{11} = stiffness coefficient

$$Q_{11} = \frac{1 - \mu_{23}\mu_{32}}{E_2 E_3 \Delta}$$

$$Q_{12} = \frac{\mu_{21} + \mu_{23}\mu_{31}}{E_2 E_3 \Delta}$$

$$Q_{13} = \frac{\mu_{31} + \mu_{21}\mu_{32}}{E_1 E_3 \Delta}$$

$$Q_{22} = \frac{1 - \mu_{13}\mu_{31}}{E_1 E_3 \Delta}$$

$$Q_{23} = \frac{\mu_{32} + \mu_{12}\mu_{32}}{E_1 E_3 \Delta}$$

$$Q_{33} = \frac{1 - \mu_{12}\mu_{21}}{E_1 E_2 \Delta}$$

$$Q_{66} = G_{12}$$

$$Q_{55} = G_{13}$$

$$Q_{44} = G_{23}$$

Where,

$$\Delta = \frac{[1 - \mu_{12}\mu_{21} - \mu_{23}\mu_{32} - \mu_{13}\mu_{31} - 2[\mu_{21}\mu_{32}\mu_{13}]]}{E_1 E_2 E_3}$$

Governing Equation

By using the principal of virtual work

Internal force = External force

$$\int_{x=0}^a \int_{y=0}^b \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_{xx} \delta \epsilon_{xx} + \sigma_{yy} \delta \epsilon_{yy} + \sigma_{zz} \delta \epsilon_{zz} + \tau_{xz} \delta \vartheta_{zx} + \tau_{xy} \delta \vartheta_{xy} + \tau_{yz} \delta \vartheta_{yz}] dx dy dz = \int_0^a \int_0^b \partial \omega dx dy = 0$$

$$\int_{x=0}^a \int_{y=0}^b \int_{-h/2}^{h/2} \left[\sigma_{xx} \delta \left(\frac{\partial u_0}{\partial x} - z \frac{\partial^2 \omega_0}{\partial x^2} + f(z) \frac{\partial \phi_0}{\partial x} \right) + \sigma_{yy} \delta \left(\frac{\partial v_0}{\partial y} - z \frac{\partial^2 \omega_0}{\partial y^2} + f(z) \frac{\partial \psi_0}{\partial y} \right) + \sigma_{zz} \delta(0) + \tau_{xz} \frac{\partial f(z)}{\partial z} \phi_0 + \tau_{xy} \frac{\partial u_0}{\partial y} + f(z) \frac{\partial \phi_0}{\partial y} + \frac{\partial v_0}{\partial x} + f(z) \frac{\partial \psi_0}{\partial x} - 2z \frac{\partial^2 \omega_0}{\partial y \partial x} + \tau_{yz} \frac{\partial f(z)}{\partial z} \psi_0 \right] dx dy dz \quad (A)$$

Assume,

Axially loading along the direction

$$N_x = \int_{z=-h/2}^{h/2} \sigma_{xx} dz$$

$$N_y = \int_{z=-h/2}^{h/2} \sigma_{yy} dz$$

$$N_{xy} = \int_{z=-h/2}^{h/2} \tau_{xy} dz$$

Bending moment

$$M_x^b = \int_{z=-h/2}^{h/2} \sigma_{xx} z dz$$

$$M_y^b = \int_{z=-h/2}^{h/2} \sigma_{yy} z dz$$

$$M_{xy}^b = \int_{z=-h/2}^{h/2} \tau_{xy} z dz$$

Shear or twisting moment

$$M_x^s = \int_{z=-h/2}^{h/2} \sigma_{xx} f(z) dz$$

$$M_y^s = \int_{z=-h/2}^{h/2} \sigma_{yy} f(z) dz$$

$$M_{xy}^s = \int_{z=-h/2}^{h/2} \tau_{xy} f(z) dz$$

Shear stress

$$Q_x = \int_{z=-h/2}^{h/2} \tau_{xz} \frac{\partial f(z)}{\partial z} dz$$

$$Q_y = \int_{z=-h/2}^{h/2} \tau_{yz} \frac{\partial f(z)}{\partial z} dz$$

$$Q_z = 0$$

$$\phi = \text{coefficient factor}$$

∴ shear stress w.r. to z only

$$\int_{x=0}^a \int_{y=0}^b \int_{-h/2}^{h/2} \left(N_x \delta \frac{\partial u_0}{\partial x} - M_x^b \delta \frac{\partial^2 \omega_0}{\partial x^2} + M_x^s \delta \frac{\partial \phi_0}{\partial x} \right) + \left(N_y \delta \frac{\partial v_0}{\partial y} - M_y^b \delta \frac{\partial^2 \omega_0}{\partial y^2} + M_y^s \delta \frac{\partial \psi_0}{\partial y} \right) + 0 + Q_x * 0 + N_{xy} \delta \frac{\partial u_0}{\partial y} + M_{xy}^s \delta \frac{\partial \phi_0}{\partial y} + N_{xy} \delta \frac{\partial v_0}{\partial x} + M_{xy}^s \delta \frac{\partial \psi_0}{\partial x} - M_{xy}^b \delta 2 \left(\frac{\partial^2 \omega_0}{\partial y \partial x} \right) dx dy dz$$

Illustrative Examples

To assess the performance of present theory in the prediction of thermal and thermo-mechanical response of laminates under single sinusoidal and uniformly distributed thermal load, as well as under uniformly distributed thermo-mechanical load, various are considered here in.

The material properties used are as follows:

Material I

Isotropic material

$$E_c = 380 \text{ GPa}, \mu = 0.3, \alpha_c = 7.4 \times 10^{-6}$$

Material II

Isotropic material

$$E = 210, \mu = 0.3, G = \frac{E}{2(1+\mu)} \quad \alpha_c = 3,$$

Material III

Orthotropic material

$$\frac{E_1}{E_2} = 25, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2,$$

$$\mu_{12} = 0.25, \mu_{21} = \frac{E_2}{E_1} \mu_{12}, \frac{\alpha_y}{\alpha_x} = 3$$

α_x and α_y are coefficients of thermal expansions in the direction of fibre and in transverse direction respectively.

Problems Based on Linear Thermal Load

Example 1

A simply supported square isotropic plate subjected to single sinusoidal linear thermal load $T(x, y, z) = \frac{2}{h} z T_1(x, y)$ across the thickness of plate is considered to evaluate h displacements and stresses. (Material I)

Example 2

A simply supported square isotropic plate subjected to uniform linear thermal load $T(x, y, z) = z T_1(x, y)$ across the thickness of plate is considered to evaluate displacements and stresses. (Material II)

Example 3

A simply supported square isotropic plate subjected to uniform linear thermal load $T(x, y, z) = z T_1(x, y)$ across the thickness of plate and uniform transverse mechanical load is considered to evaluate displacements and stresses. (Material II)

Example 4

A simply supported rectangular isotropic plate subjected to single sinusoidal linear thermal load $T(x, y, z) = z T_1(x, y)$ across the thickness of plate is considered to evaluate transverse displacements. (Material II)

Example 5

A simply supported rectangular isotropic plate subjected to uniform linear thermal load $T(x, y, z) = z T_1(x, y)$ across the thickness of plate is considered to evaluate transverse displacements. (Material II)

Example 6

A simply supported square orthotropic plate subjected to single sinusoidal linear thermal load $T(x, y, z) = z T_1(x, y)$ through the thickness of plate is considered to evaluate the thermal response (displacements and stresses). (Material III)

Example 7

A simply supported rectangular orthotropic plate subjected to single sinusoidal linear thermal load $T(x, y, z) = z T_1(x, y)$ across the thickness of plate is considered to evaluate transverse displacements for various ratios of plate dimensions. (Material III)

The Solution Scheme

Here we concern with the close form solutions of simply supported square and rectangular plates. The boundary conditions for simply supported edges are

$$\begin{aligned} v_0 = \omega = \psi = N_x = M_x = M_x^S = 0 \quad x = 0 \text{ to } x = a \\ u_0 = \omega = \phi = N_y = M_y = M_{xy}^S = 0 \quad y = 0 \text{ to } y = b \end{aligned}$$

The assumed analytical solution is in the form of double trigonometric series, which satisfies the governing equations and boundary conditions exactly. Such solution in theory of plate and shells is called as closed formed solution. This type of solution was suggested Navier (1820) for the bending problem of simply supported rectangular plates. The uniform transverse mechanical load and uniform in thermal load are also expressed in terms of document Fourier trigonometric sine series according to Navier solution procedure.

The following is the solution form for $u_0(x, y)$, $v_0(x, y)$, $\omega(x, y)$, $\phi(x, y)$, $\psi(x, y)$

Satisfies above boundary conditions exactly;

$$\begin{cases} u_0 \\ v_0 \\ \omega \\ \phi \\ \psi \end{cases} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{cases} u_{0mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ v_{0mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \\ \omega_{0mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ \phi_{0mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ \psi_{0mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \end{cases}$$

Thermal and transverse mechanical loads are expanded in double Fourier sine series

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ q \end{pmatrix} = \begin{pmatrix} T_{1mn} \\ T_{2mn} \\ T_{3mn} \\ q_{mn} \end{pmatrix} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Where,

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ q \end{pmatrix} = \frac{4}{ab} \int_0^a \int_0^b \begin{pmatrix} T_{1mn} \\ T_{2mn} \\ T_{3mn} \\ q_{mn} \end{pmatrix} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

The coefficients T_{1mn} , T_{2mn} , T_{3mn} are defined as follows.

$T_{1mn} = T_{2mn} = T_{3mn} = T_0$ for $m = n = 1$ in case of single sinusoidal load

$T_{1mn} = T_{2mn} = T_{3mn} = \frac{16 T_0}{\pi^2 mn}$ for m, n is odd; uniformly distributed transverse load, T_0 Represents intensity of the thermal load.

$q_0 = q_{mn}$ For $m = n = 1$ in case of single sinusoidal load.

$q_{mn} = \frac{16 T_0}{\pi^2 mn}$ for m, n is odd; uniformly distributed transverse load, q_0 Represents intensity of the transverse load.

Substitution of solution form given by equations into governing equations results into a system of the algebraic equations which can be written into a matrix form as follows:

$$[K]\{\delta\} = \{f\}$$

Where,

$[K]$ is the symmetric stiffness matrix, $\{\delta\} = \{u_{0mn}, v_{0mn}, \omega_{mn}, \phi_{mn}, \psi_{mn}\}^T$ and $\{f\}$ is the generalized force vector.

From solution of these equations, unknown coefficients, $\{\delta\}$ can be obtained readily. Substituting these coefficients into equations generalized displacements and rotations can be obtained and subsequently in-plane stresses and transverse stresses can be obtained. Although the stress components can be calculated from the constitutive relations, these stresses may not satisfy the continuity conditions at the interface between layers. Hence transverse shear stresses in orthotropic, symmetric and anti-symmetric cross-ply laminated plates are obtained by using three-dimensional stress equilibrium equations of elasticity.

PERFORMANCE ANALYSIS

Introduction

In this chapter, displacements and stresses are determined for isotropic, orthotropic, anti-symmetric and symmetric laminated plates subjected to linear and non-linear thermal loads through the thickness of plate in combination with transverse mechanical load. The results obtained by present theory are compared and discussed with the corresponding results of classical plate theory [1, 2], first-order shear deformation theory [10] and higher-order shear deformation theory of Reddy [26]. It is to note that the complete results of displacements and stresses are specially generated using above theories for the purpose of comparison and discussion being not available in the literature and are presented in following normalized form.

In case of linear and non-linear thermal loads, following normalized forms are used:

$$\begin{aligned}\bar{u} &= u \left(0, \frac{b}{2}, \frac{h}{2}\right) \frac{1}{T_0 \alpha_1 a}, \bar{v} = v \left(\frac{a}{2}, 0, \frac{h}{2}\right) \frac{1}{T_0 \alpha_1 a}, \bar{\omega} = \omega \left(\frac{a}{2}, \frac{b}{2}, 0\right) \frac{10 \times h}{T_0 \alpha_1 b^2} \\ (\bar{\sigma}_x, \bar{\sigma}_y) &= (\sigma_x, \sigma_y) \left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right) \frac{1}{2 T_0 \alpha_1 b E_2}, \bar{\tau}_{xy} = \tau_{xy} \left(0, 0, -\frac{h}{2}\right) \frac{1}{T_0 \alpha_1 b E_2}, \\ \bar{\tau}_{xz} &= \tau_{xz} \left(0, \frac{b}{2}, -\frac{h}{6}\right) \frac{1}{T_0 \alpha_1 b E_2}, \bar{\tau}_{yz} = \tau_{yz} \left(\frac{a}{2}, 0, -\frac{h}{6}\right) \frac{1}{T_0 \alpha_1 b E_2}\end{aligned}$$

In case thermo-mechanical loads, following normalized forms are used:

Linear Thermal Load

Numerical Results for Isotropic Plate

The results obtained for displacements and stresses for isotropic plate are presented in the tables as given below Table 1–5.

Discussion

Numerical results for displacements and stresses for square isotropic plate are presented in above Tables 1 through 45. It is seen that for isotropic plate, results obtained by present theory for displacements and stresses are same as that of HSDT, FSDT and CPT under single sinusoidal and uniform thermal loads. It is observed that, transverse shear stresses obtained by present theory, HSDT, FSDT and CPT under sinusoidal and uniform thermal load are zero, whereas transverse shear stresses are not zero under uniform thermo-mechanical loads. This shows the effect of mechanical load on isotropic plate. Effect of ratio of plate dimension and side-to-thickness ratio on transverse deflection under sinusoidal and uniform thermal load is shown in Tables 4 and 5. It is observed that, as the ratio of plate dimension increase transverse deflection also increases.

Table 1. Normalized displacements and stresses for square isotropic plate subjected to single sinusoidal linear thermal load for aspect ratio 10. (Example 1)

Theory	$\bar{u}$	$\bar{v}$	$\bar{\omega}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{EE}$	$\bar{\tau}_{zy}^{EE}$
Present	2.0690	2.0690	13.1718	0.500	0.5000	0.5000	0.00	0.000
HSDT (Reddy J.N.) 1980	2.0690	2.0690	13.1718	0.500	0.5000	0.5000	0.00	0.000
FSDT (Mindlin R.D.) 1951	2.0690	2.0690	13.1717	0.500	0.5000	0.5000	0.00	0.000
CPT (Kirchhoff G.R.) 1850	2.0690	2.0690	13.1718	0.500	0.5000	0.5000	0.00	0.000

Table 2. Normalized displacements and stresses for square isotropic plate subjected to uniform linear thermal load for aspect ratio 10. (Example 2)

Theory	$\bar{u}$	$\bar{v}$	$\bar{\omega}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{EE}$	$\bar{\tau}_{zy}^{EE}$
Present	0.0213	0.0213	0.9575	0.0234	0.0234	0.1120	0.00	0.00
HSDT (Reddy J.N.) 1980	0.0213	0.0213	0.9575	0.0234	0.0234	0.1120	0.00	0.00
FSDT (Mindlin R.D.) 1951	0.0213	0.0213	0.9575	0.0234	0.0234	0.1120	0.00	0.00
CPT (Kirchhoff G.R.) 1850	0.0213	0.0213	0.9575	0.0234	0.1120	0.1120	0.00	0.00

Table 3. Normalized displacements and stresses for square isotropic plate subjected to uniform linear thermal load across the thickness of plate and uniform transverse mechanical load for aspect ratio 10. (Example 3)

Theory	$\bar{u}$	$\bar{v}$	$\bar{\omega}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{EE}$	$\bar{\tau}_{zy}^{EE}$
Present	0.0384	0.0384	5.5175	-0.0774	-0.0774	0.1401	0.2153	0.2153
HSDT (Reddy J.N.) 1980	0.0384	0.0384	5.4982	-0.0775	-0.0775	0.1395	0.2161	0.2161
FSDT (Mindlin R.D.) 1951	0.0382	0.0382	5.5187	-0.0768	-0.0768	0.1386	0.2163	0.2163
CPT (Kirchhoff G.R.) 1850	0.0382	0.0382	5.3280	-0.0768	-0.0768	0.1386	0.2163	0.2163

Table 4. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection W_i of simply supported rectangular isotropic plate subjected to single sinusoidal linear thermal load for aspect ratio 10. (Example 4)

Theory	a/b = 1	a/b = 1.5	a/b = 2	a/b = 2.5	a/b = 3
Present	0.6586	0.9119	1.0537	1.1355	1.1855
HSDT (Reddy J.N.) 1980	0.6586	0.9119	1.0537	1.1355	1.1855
FSDT (Mindlin R.D.) 1951	0.6586	0.9117	1.0541	1.1361	1.1857
CPT (Kirchhoff G.R.) 1850	0.6586	0.9119	1.0537	1.1355	1.1855

Table 5. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{\omega}$ of simply supported rectangular isotropic plate subjected to uniform linear thermal load for aspect ratio 10. (Example 5)

Theory	a/b = 1	a/b = 1.5	a/b = 2	a/b = 2.5	a/b = 3
Present	0.9575	1.3097	1.4798	1.5582	1.5938
HSDT (Reddy J.N.) 1980	0.9575	1.3097	1.4798	1.5582	1.5938
FSDT (Mindlin R.D.) 1951	0.9575	1.3095	1.4792	1.5582	1.5942
CPT (Kirchhoff G.R.) 1850	0.9575	1.0397	1.4798	1.5582	1.5938

Numerical Results for Orthotropic Plate:

The results obtained for displacements and stresses are presented in the tables as given below Table 6 and 7.

Table 6. Normalized displacements and stresses for square orthotropic plate subjected to single sinusoidal linear thermal load across the thickness of plate. (Example 6)

S	Theory	$\bar{u}$	$\bar{v}$	$\bar{\omega}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{EE}$	$\bar{\tau}_{zy}^{EE}$
5	Present	0.3222	0.3729	1.0709	0.0155	0.01830	0.1092	0.1017	0.1100
	HSDT (Reddy J.N.) 1980	0.3221	0.3489	1.0711	0.0179	0.1906	0.1054	0.1290	0.1293
	FSDT (Mindlin R.D.) 1951	0.3190	0.3812	1.0721	0.0394	0.1806	0.1100	0.1108	0.1108
	CPT (Kirchhoff G.R.) 1850	0.3240	0.3240	1.0312	0.0053	0.1983	0.1018	0.1516	0.1516
10	PRESENT	0.1617	0.1697	1.0439	0.0042	0.0967	0.0521	0.0340	0.0346
	HSDT (Reddy J.N.) 1980	0.1617	0.1675	1.0439	0.0047	0.0974	0.0517	0.0354	0.0355
	FSDT (Mindlin R.D.) 1951	0.1612	0.1709	1.0440	0.0080	0.0964	0.0522	0.0347	0.0347
	CPT (Kirchhoff G.R.) 1850	0.1620	0.1620	1.0312	0.0026	0.0991	0.0509	0.0379	0.0379

Table 7. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{\omega}$ of simply supported rectangular orthotropic plate subjected to single sinusoidal linear thermal load. (Example 7)

Theory	b/h	a/b = 1	a/b = 1.5	a/b = 2	a/b = 2.5	a/b = 3
Present	10	1.0439	2.1127	3.0804	3.6617	3.9042
HSDT (Reddy J.N.) 1980	10	1.0358	2.1127	3.0804	3.6617	3.9042
FSDT (Mindlin R.D.) 1951	10	1.0440	2.1126	3.0803	3.6613	3.9038
CPT (Kirchhoff G.R.) 1850	10	1.0312	2.1127	3.0806	3.6619	3.9044
PRESENT	20	1.0346	2.1127	3.0805	3.6619	3.9044
HSDT (Reddy J.N.) 1980	20	1.0336	2.1127	3.0805	3.6619	3.9044
FSDT (Mindlin R.D.) 1951	20	1.0346	2.1123	3.0799	3.6612	3.9013
CPT (Kirchhoff G.R.) 1850	20	1.0312	2.1127	3.0806	3.6619	3.9044
PRESENT	100	1.0313	2.1127	3.0806	3.6619	3.9044
HSDT (Reddy J.N.) 1980	100	1.0313	2.1127	3.0806	3.6619	3.9044
FSDT (Mindlin R.D.) 1951	100	1.0313	2.1271	3.1202	3.7235	3.9366
CPT (Kirchhoff G.R.) 1850	100	1.0312	2.1127	3.0806	3.6619	3.9044

Discussion

(Orthotropic plate subjected to single sinusoidal thermal load)

The results obtained for displacements and stresses are presented in Tables 6 and 7.

In-plane Displacements ($\bar{u}, \bar{v}$):

In-plane displacements obtained by present theory, HSDT, FSDT and CPT are presented in Table 7 for aspect ratio 5 and 10. The coefficient of thermal expansion in the direction of fibre is smaller than in t displacement in the direction fibre is less than transverse direction. Hence, in-plane the displacement in transverse direction. Variation of in-plane displacements($\bar{u}$) and($\bar{v}$) through the thickness of orthotropic plate is shown in Figures. 2 to 5 respectively.

Transverse Displacements ($\bar{\omega}$):

The transverse displacements for aspect ratio 5 and 10 are presented in Table 6. The examination of the maximum deflection obtained by the present theory is in good agreement with higher-order shear deformation theory and first-order shear deformation theory for aspect ratio 5 and 10. It is observed that as aspect ratio increases from 5 to 10, transverse displacement decreases by present theory, HSDT and FSDT. However, classical plate theory does not show any change in transverse displacement as aspect ratio changes from 5 to 10.

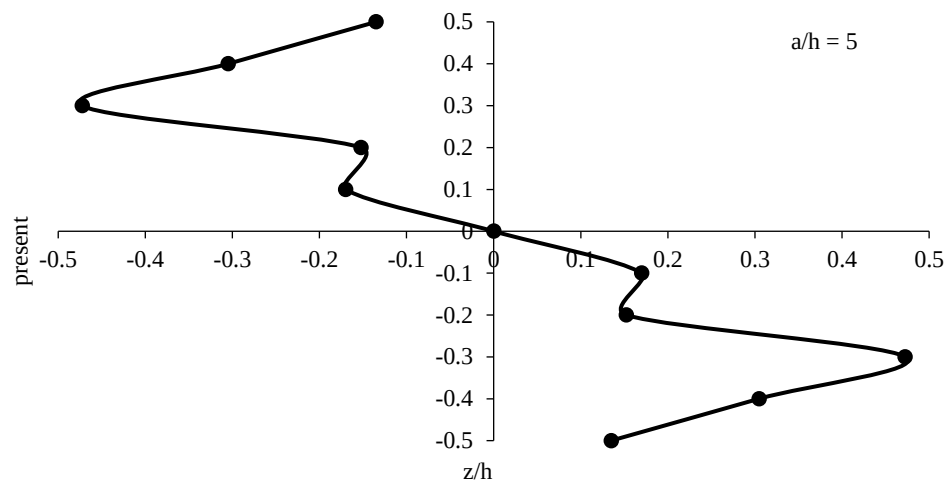


Figure 2. Variation of normalized in-plane displacement $\bar{u}$ through the thickness of orthotropic plate for aspect ratio 5.

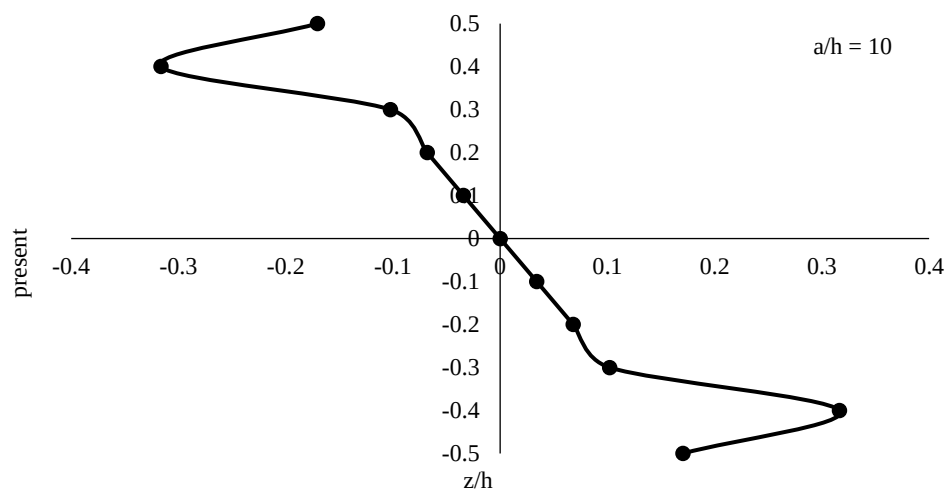


Figure 3. Variation of normalized in-plane displacement $\bar{u}$ through the thickness of orthotropic plate for aspect ratio 10.

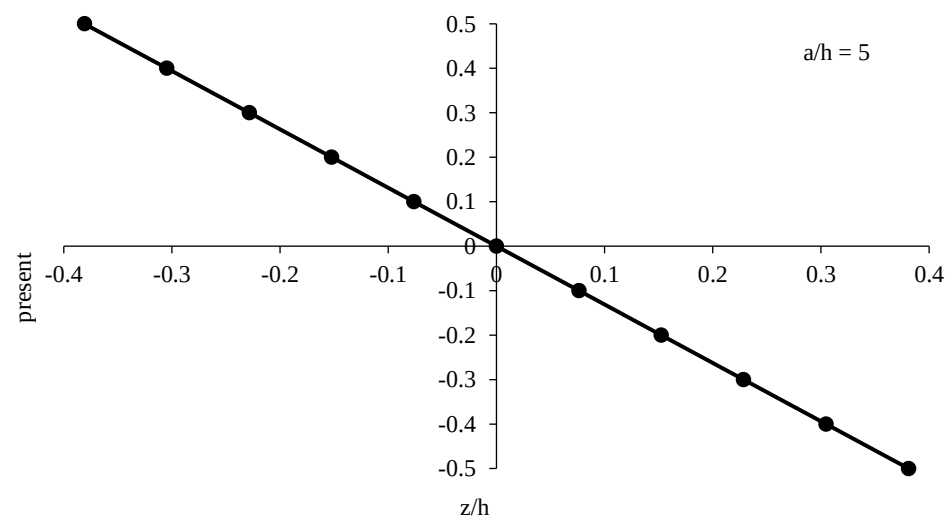


Figure 4. Variation of normalized in-plane displacement $\bar{v}$ through the thickness of orthotropic plate for aspect ratio 5.

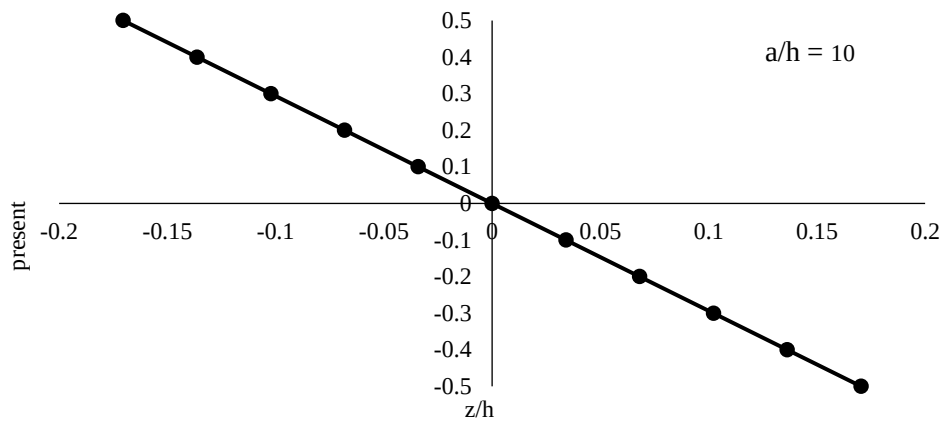


Figure 5. Variation of normalized in-plane displacement $\bar{v}$ through the thickness of orthotropic plate for aspect ratio 10.

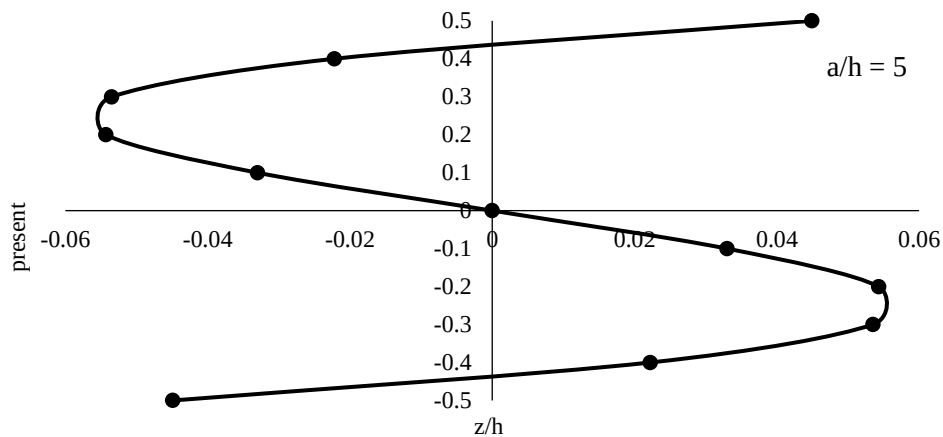


Figure 6. Variation of normalized in-plane normal stress $\bar{\sigma}_x$ through the thickness of orthotropic plate for aspect ratio 5.

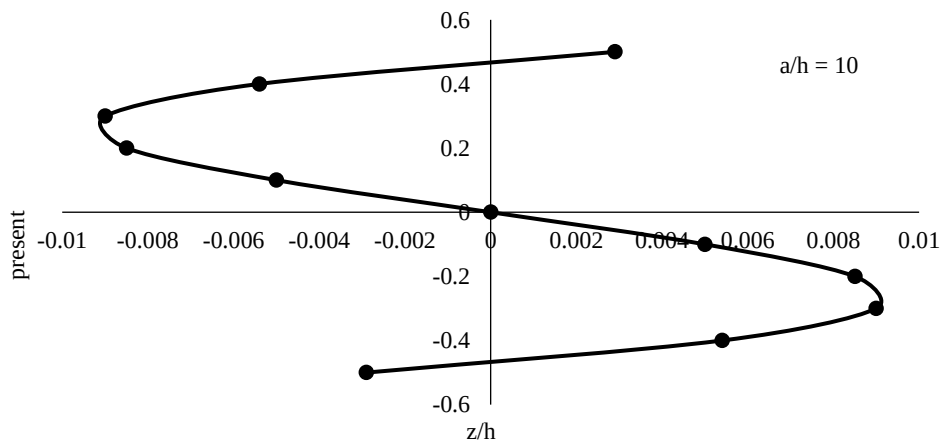


Figure 7. Variation of normalized in-plane normal stress $\bar{\sigma}_x$ through the thickness of orthotropic plate for aspect ratio 10.

In-plane Normal and Shear Stresses ($\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}$)

In-plane normal and shear stresses are presented in Table 6 under single sinusoidal thermal load for aspect ratio 5 and 10. It is observed that, in-plane stress in the direction of fibre is less and in-plane stress in transverse direction is more. Figures.6 to 11 shows curvilinear variation of normal stress σ_x

through the thickness of orthotropic plate by present theory and HSDT indicating the shear deformation effect, but those of CPT and FSDT are linear for aspect ratios 5 and 10 respectively. Figures 6 to 11 show linear variation of in-plane normal stress σ_y and in-plane shear τ_{xy} through the thickness of orthotropic plate.

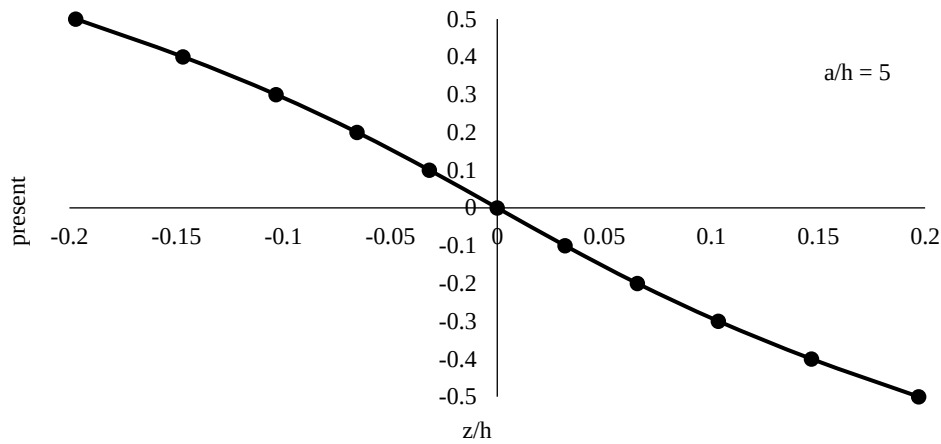


Figure 8. Variation of normalized in-plane normal stress $\overline{\sigma_y}$, through the thickness of orthotropic plate for aspect ratio 5.

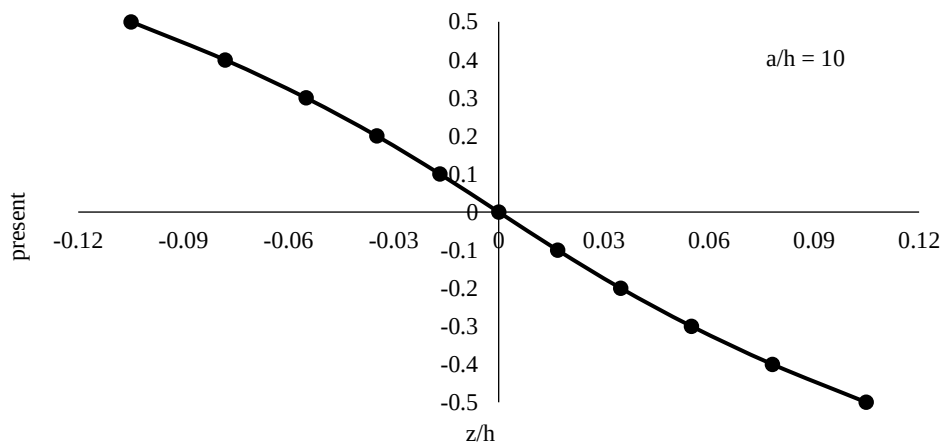


Figure 9. Variation of normalized in-plane normal stress $\overline{\sigma_y}$, through the thickness of orthotropic plate for aspect ratio 10.

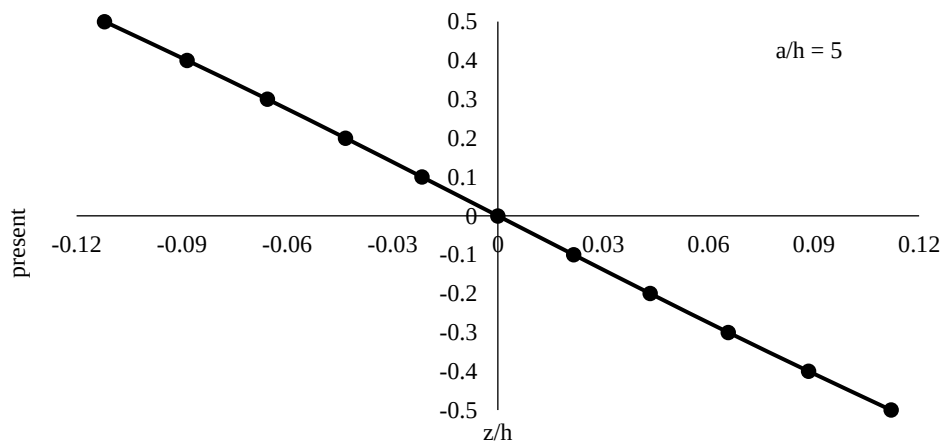


Figure 10. Variation of normalized in-plane shear stress $\overline{\tau_{xy}}$ through the thickness of orthotropic plate for aspect ratio 5.

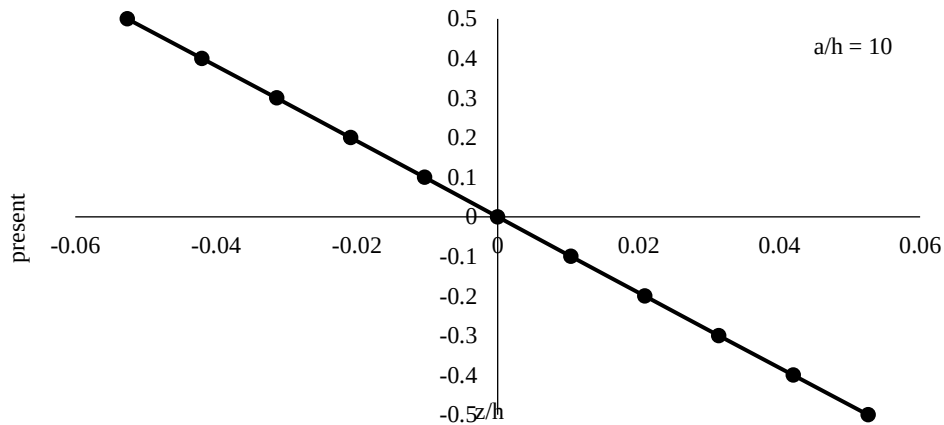


Figure 11. Variation of normalized in-plane shear stress $\bar{\tau}_{xy}$ through the thickness of orthotropic plate for aspect ratio 10.

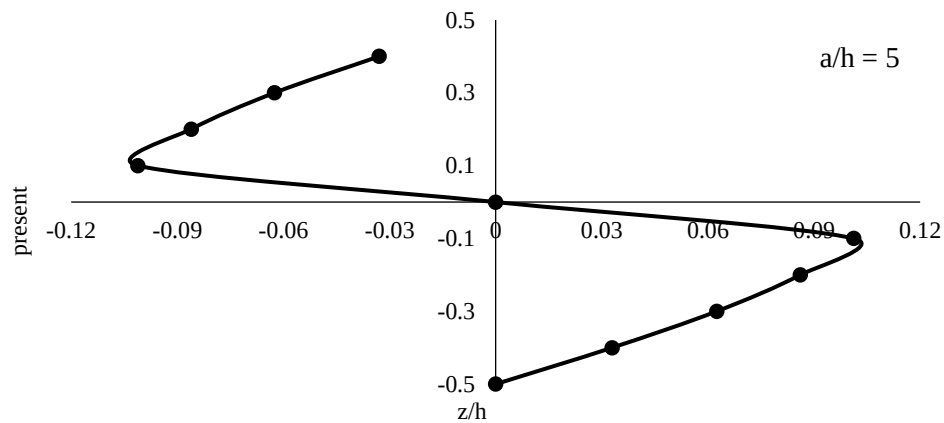


Figure 12. Variation of normalized transverse stress $\bar{\tau}_{zx}$ through the thickness of orthotropic plate for aspect ratio 5.

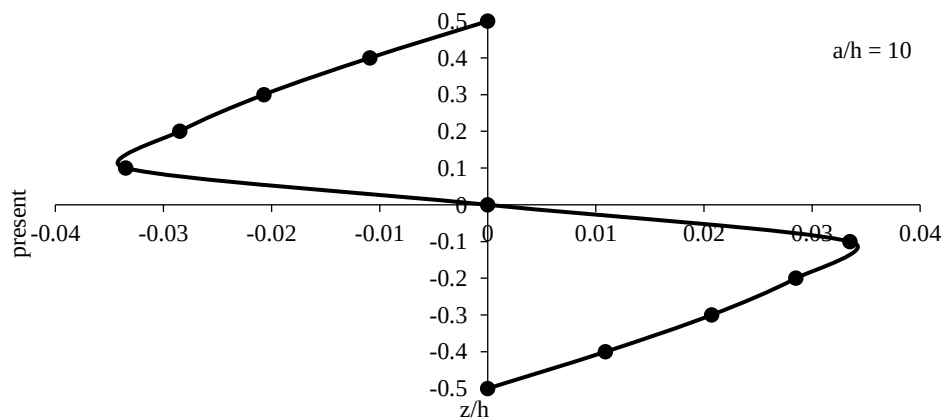


Figure 13. Variation of normalized transverse stress $\bar{\tau}_{zx}$ through the thickness of orthotropic plate for aspect ratio 10.

Transverse Shear Stresses ($\bar{T}_{zx}$, $\bar{T}_{zy}$)

Transverse shear stresses obtained by present theory, HSDT, FSDT and CPT are presented in Table 6 for aspect ratio 5 and 10. Transverse shear stresses are obtained by integration of equilibrium equations of three-dimensional elasticity is shown in Figures 12 to 15. The transverse shear stress satisfies the stress-free boundary conditions at the top and bottom of the plate which obviate the need of shear correction factor.

Transverse shear stresses obtained by present theory, are in good agreement with HSDT and FSDT, whereas CPT over predicts the transverse shear stresses.

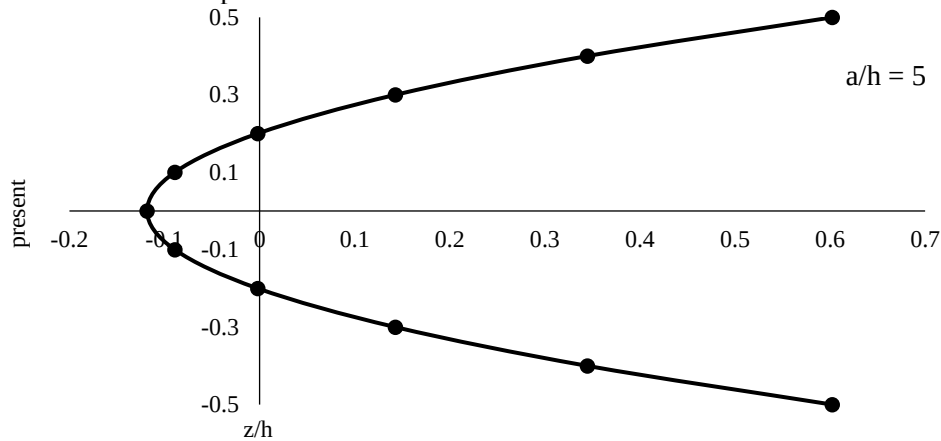


Figure 14. Variation of normalized transverse stress $\bar{\tau}_{zy}$ through the thickness of orthotropic plate for aspect ratio 5.

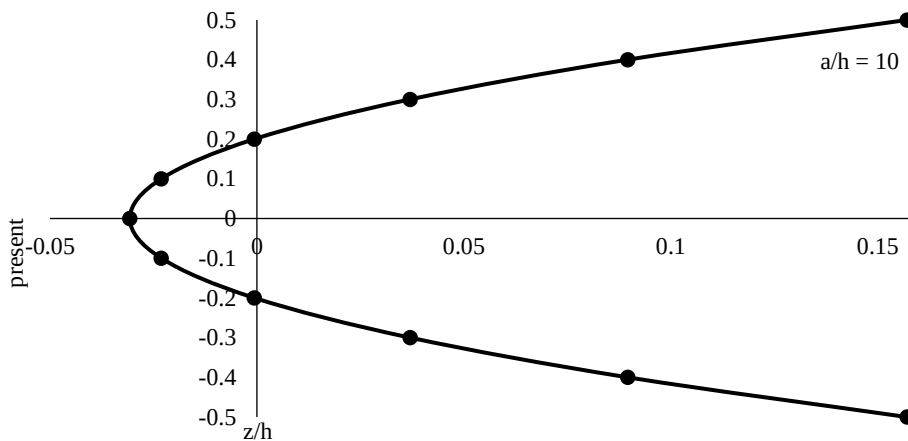


Figure 15. Variation of normalized transverse stress $\bar{\tau}_{zy}$ through the thickness of orthotropic plate for aspect ratio 10.

Effect of Ratio of Plate Dimension

The transverse displacements for rectangular plate are presented in Table 7 and Figures (16 to 20). It is observed from the results that as aspect ratio of plate dimension increases transverse displacement also increases.

$$\bar{u} = u\left(0, \frac{b}{2}, \frac{h}{2}\right) \frac{1}{\left(q_0 h s^3 / E_2\right) + (T_0 \alpha_1 a)}, \quad \bar{v} = v\left(\frac{a}{2}, 0, \frac{h}{2}\right) \frac{1}{\left(q_0 h s^3 / E_2\right) + (T_0 \alpha_1 a)},$$

$$\bar{\omega} = \omega\left(\frac{a}{2}, \frac{b}{2}, 0\right) \frac{100 \times h}{\left(q_0 a^4 / E_2 h^3\right) + (T_0 \alpha_1 a^2) / 10h},$$

$$(\bar{\sigma}_x) = (\sigma_x)\left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right) \frac{1}{q_0 a^2 / h^2 + (T_0 \alpha_1 b E_2)},$$

$$\bar{\sigma}_y = \sigma_y\left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right) \frac{\sigma_y}{q_0 a^2 / h^2 + (T_0 \alpha_1 b E_2)},$$

$$\bar{\tau}_{xy} = \tau_{xy}\left(0, 0, -\frac{h}{2}\right) \frac{1}{q_0 a^2 / h^2 + (T_0 \alpha_1 b E_2)},$$

$$\overline{\tau_{xz}} = \tau_{xz} \left(0, \frac{b}{2}, -\frac{h}{6} \right) \frac{1}{q_0 a / h + (T_0 \alpha_1 b E_2)},$$

$$\overline{\tau_{yz}} = \tau_{yz} \left(\frac{a}{2}, 0, -\frac{h}{6} \right) \frac{1}{q_0 a / h + (T_0 \alpha_1 b E_2)}$$

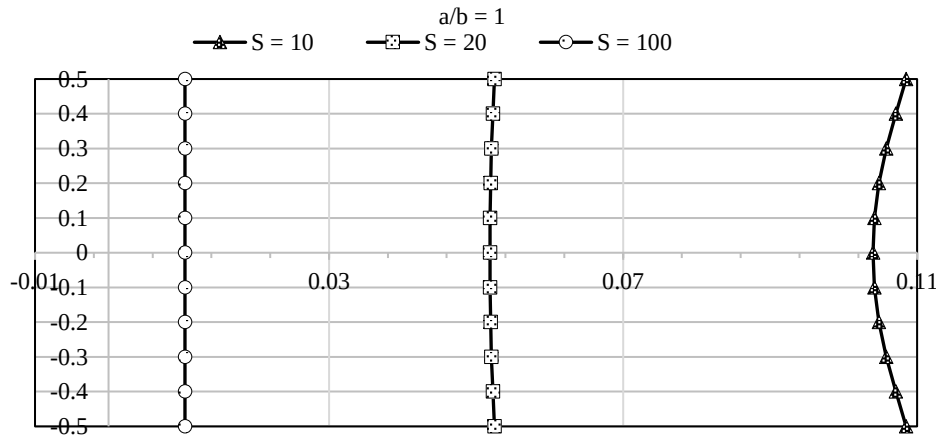


Figure 16. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{w}$ of simply supported rectangular orthotropic plate for $a/b = 1$.

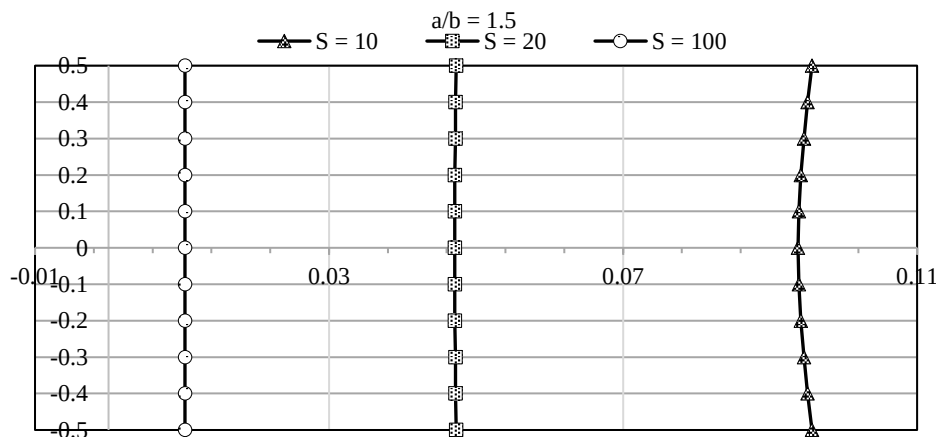


Figure 17. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{w}$ of simply supported rectangular orthotropic plate for $a/b = 1.5$.

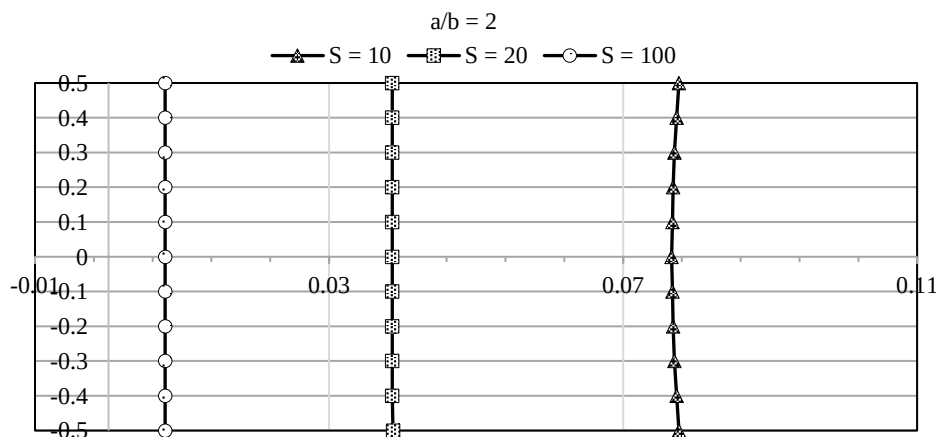


Figure 18. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{w}$ of simply supported rectangular orthotropic plate for $a/b = 2$.

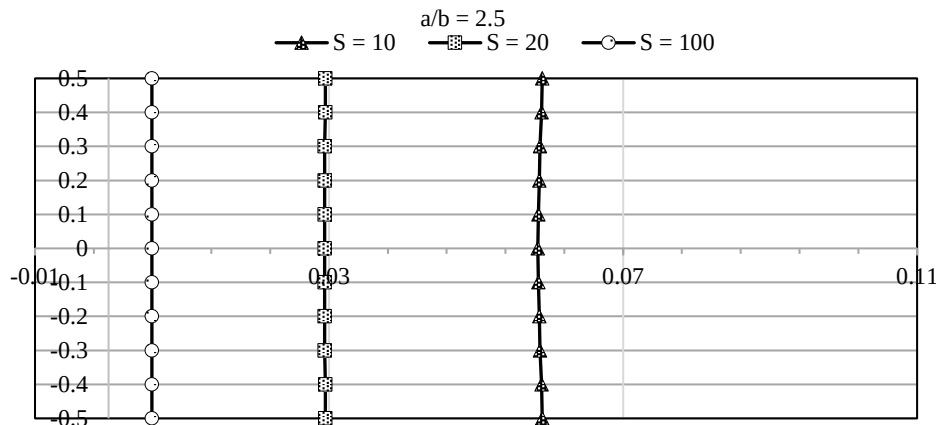


Figure 19. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{w}$ of simply supported rectangular orthotropic plate for $a/b = 2.5$.

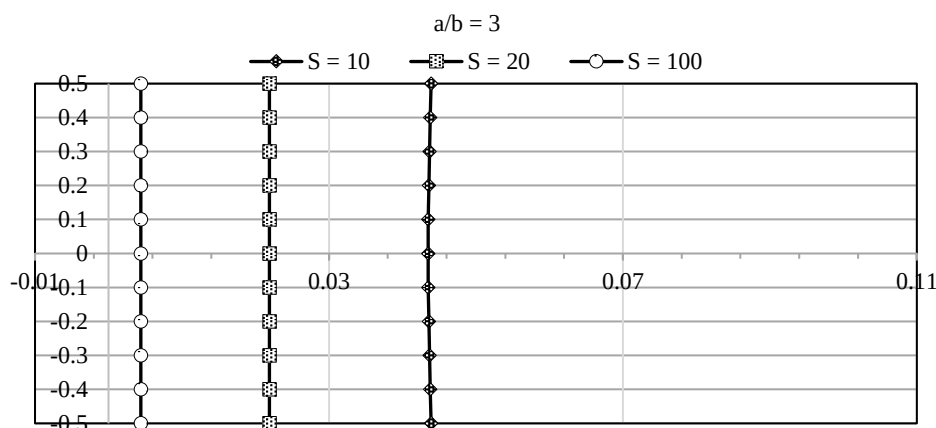


Figure 20. Effect of the ratio of plate dimension (a/b) and side-to-thickness ratio on the deflection $\bar{w}$ of simply supported rectangular orthotropic plate for $a/b = 3$.

CONCLUSIONS

A new trigonometric higher-order shear deformation theory for isotropic and laminated composite and sandwich plates is presented. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. The accuracy of the present theory is ascertained by comparing it with various available results in the literature. The results show that the present model is in close agreement with Reddy's and Touratier's shear deformation theories for analysing the static behaviour of isotropic and composite laminated and sandwich plates.

A SSNDT which included the effect of transverse shear and transverse normal strain/stress is applied to the bidirectional bending analysis of rectangular plates. The theory is variationally consistent and obviates the need of a shear correction factor due to the realistic variation of transverse shear stress through the plate thickness. The accuracy of the present theory is ascertained by comparing it with various available results in the literature which shows that, the present theory is in close agreement with higher-order shear deformation theory of Reddy and exact 3D solutions for analysing the bending response of isotropic, transversely isotropic, laminated composite and sandwich plates.

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