

Twisting, Super Twisting and Adaptive Algorithm Sliding Mode Controller for Second Order Time Delay Systems

G. Satya Rao^{1*}, B. Amarendra Reddy²

Abstract

The sliding mode control approach is widely considered as one of the most effective methods for designing a reliable controller for a complicated higher-order dynamic plant operating under uncertainty. The primary advantages of sliding mode is its low sensitivity to changes in plant parameters and the ability to entirely eliminate disruptions. Sliding mode control has been widely applied as a robust control design methodology that is completely insensitive to uncertainties, and research on sliding mode control strategy is a robust control design methodology that is completely insensitive to uncertainties is greatly motivated as challenges and demand grow. It's worth noting that a variety of enhanced sliding mode control schemes have been developed, each with its own set of desirable features. In this thesis regulation of second order plus time delay systems plays a vital role. Sliding mode control is a robust control strategy. To regulate these systems and also to improve the performance under disturbance conditions sliding mode control strategy is adapted. Pade approximations (0/1) are used for representation of the constant time delay and here (0/1) approximation is adapted. Simulation studies have been performed for these systems in the MATLAB environment and briefly presented in this work. The second order sliding mode scheme covered in this thesis are: Twisting algorithm, Super-Twisting algorithm and adaptive algorithm.

Keywords: Sliding mode control, PADE approximations, Time delay systems, PID controller, Second order sliding mode controller, Robust controller.

INTRODUCTION

The technique of time delay control [1] (TDC) has been originally developed to compensate for system uncertainties, for example, unmodeled dynamics, parameter variations, and the effect of disturbances. To estimate the effect of nonlinearities and disturbances, the approach uses historical knowledge from the control input and measurement signals.

Estimation of First-Order-Plus-Time-Delay (FOPTD) models [2] and Second-Order-Plus Time Delay (SOPTD) models [9] have received great attention due to the wide spread use of such models in

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techniques for tuning PID Controllers [3, 4]. Identification of dynamic transfer function models from experimental data is essential for model-based controller design [5]. This is one conceivable source of uncertainty and the model error is assumed to be negligible. There may be disruptions at the plant outputs, input parameter perturbations, and measurement discrepancies in real time. It's difficult to develop a controller that brings all of these factors into account. A robust controller is one that can effectively reject disturbances in the process control variables. Robust controller design strategies come in a variety of forms. Sliding mode control.

The regulation of second order plus time delay systems (SOPDT) [6], which plays a key role in many industrial applications [7] with delayed outputs in response to inputs, is explored in this paper. We discuss three crucial factors for modelling an industrial process control system [8], taking this into account for enhancing the system. There are three of them: I) gain, (ii) time-delay, and (ii) time-constant. Due to model uncertainty, or the difference between the actual plant and the model, precise depiction of such a system using these parameters is extremely challenging. From a modelling standpoint, this is one conceivable uncertainty, and the model error is assumed to be insignificant. There may be disruptions at plant outputs, input parameter fluctuations, and measurement imperfections in real time, and these are assumed to be within 15% to 20% of their respective values. Sliding mode control approach [4] is a robust control design methodology [6] that is unaffected by uncertainty. Robust controller design strategies come in a range of shapes and sizes. Sliding mode control technique is one way to robust controller design [9]. It is used to govern these systems as well as to increase performance under disturbance situations.

This paper is organized as: Significance of system parameters of SOPDT model are explained Section II. The SOSMC design procedure and control laws are discussed in section III. Simulated results for different algorithms in SOSMC are presented in section IV and conclusions from results are drawn in section V.

SIGNIFICANCE OF TIME CONSTANTS, GAIN AND TIME-DELAY OF SOPDT MODEL

Consider an industrial process control system which is described using SOPDT model and its transfer function is given in equation (1).

$$G(s) = \frac{Ke^{-\alpha_d s}}{(1+\alpha_1 s)(1+\alpha_2 s)} \quad (1)$$

Usually study of system gain K and time constants α_1 , α_2 and delay time α_d helps to understand the model behaviour. For a certain change in controller output, the response of process variable behaves according to the change in gain.

The SOPDT model describes a process with two time constants, which can be used to model plant behaviour. The dead-time specifies how much delay is expressed in the output for a change in controller output, although it has no effect on the response's nature. Because it has fewer characteristics, the frequency response of the FOPDT model does not match that of the actual plant at specified corner frequencies. SOPDT representations can be used to reduce frequency response mismatch. As a result, it is preferable to use the SOPDT model rather than the FOPDT model. The time-delay of the SOPDT model is represented using (0/1) Pade-approximation and it is described using equation 2.

$$e^{-\alpha s} = \frac{1}{1+\alpha s} \quad (2)$$

The transfer function of SOPDT model using Pade approximation is given using equation (2).

Using phase variable canonical state space model, the eqn. (3) is expressed as given in eqn. (3).

$$\dot{X} = AX + BU \quad (3)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -p_1 & -p_2 & -p_3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$$

Where

$$p_1 = \frac{1}{\alpha_1 \alpha_2 \alpha_d}, p_2 = \frac{\alpha_1 + \alpha_2 + \alpha_d}{\alpha_1 \alpha_2 \alpha_d}, p_3 = \frac{\alpha_1 \alpha_2 + \alpha_2 \alpha_d + \alpha_1 \alpha_d}{\alpha_1 \alpha_2 \alpha_d}$$

Where $X_1(t)$ is the output, $u(t)$ is the control input and p_1 , p_2 , p_3 are functions of plant parameters.

SECOND ORDER SLIDING MODE CONTROL (SOSMC) OF A SOPDT MODEL

The sliding mode approach is an effective tool for designing robust controllers for complex higher order dynamic planes that operate under disturbance or uncertainty. The advantages of using a sliding mode control strategy. The design of a second-order sliding mode controller entails the creation of a switching surface as well as a controller. Designing a surface/plane's' that is a linear combination of the state variables is known as switching surface design. The SOSMC controller's job is to keep the variables on this surface while also making the system sensitive to uncertainties. If the state variables deviate from the specified switching surface, the controller must bring them back to the surface. The switching surface is defined by equations $s = \dot{s} = 0$

- For robust performance, the parameter uncertainties can be adaptively predicted and combined with SMC. The rigorous criteria for steady state output of a second order sliding mode surface less settling time
- Small peak overshoot. This can be solved with dynamic sliding surfaces, with both sliding mode and second order sliding mode surfaces making significant contributions. Dynamic sliding surfaces are built with variable damping ratios and settling times to increase dynamic performance.

Defining a Sliding Surface

In SOSM control approach the history of all state variables will converge to an equilibrium point when they obey $s = \dot{s} = 0$ of the state plane. The switching surface of the SOSMC strategy is stated using the equation (4).

$$s = \{X \in R^K; s = \dot{s} = 0\} \quad (4)$$

The mathematical representation of switching surface is stated as $s(x)=Sx$, here x is a state vector and $s \in R^n$ is switching surface vector. The switching functions and it's first time derivative with respect to time is continuous and its second time derivative w.r.t time is discontinuous.

$$\dot{s} = \frac{\partial s}{\partial x} \dot{x} = \frac{\partial s}{\partial x} (f(x) + b(x)u) \quad (5)$$

Differentiating twice the switching variable "s" produces a relation as given in equation (5).

$$\ddot{s} = \frac{\partial \dot{s}}{\partial x} (f(x) + b(x)u) + \frac{\partial \dot{s}}{\partial u} \dot{u} \quad \ddot{s} = \alpha(x) + \beta(x)\dot{u} \quad (6)$$

If $|s(x)| < s_0$, the following inequalities are assumed.

$$|\alpha(x)| < \phi, 0 < \Gamma_m \leq \beta(x) \leq \Gamma_M, \beta(x) = \frac{\partial \dot{s}}{\partial u}$$

The SOPDT system is represented in the regular form and the respective matrices are obtained as given.

$$A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ A_{21} = [-a_1 \quad -a_2] \quad A_{22} = [-a_3]$$

The switching surface design involves the design of state feedback matrix M such that the eigenvalues of $(A_{11}-A_{12}M)$ are in the left half of s-plane and to achieve this the desired characteristic equation is needed. It is the second order quadratic equation as given in eqn. (7)) where parameters ξ and ω_n represent the desired damping ratio and natural frequency respectively. The M matrix is obtained by comparing equation (7) with $(A_{11}-A_{12}M)$ and it is given

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0 \quad (7)$$

$$M = [m_1 \quad m_2] = [\omega_n^2 \quad 2\xi\omega_n]$$

The sliding surface is given by

$$s = [m_1 \quad m_2 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$s = m_1 x_1(t) + m_2 x_2(t) + x_3(t)$$

Where,

$$m_1 = \omega_n^2, m_2 = 2\xi\omega_n.$$

The time derivative of the switching surface (first order) is given

$$\dot{s}(t) = -p_1 x_1(t) + (m_1 - p_2)x_2(t) + (m_2 - p_3)x_3(t) + bu(t)$$

Control Law

The control laws for SOPDT system using twisting, super-twisting and adaptive algorithms are described in the following paragraphs.

Twisting Algorithm Second Order Plus Delay (SOPDT) Model

Using twisting algorithm the trajectories are plotted using s and $\dot{s}$ dot are twisting around the origin. In the twisting algorithm of the second order sliding mode control assumes that after a finite time interval the point $s = \dot{s} = 0$ will be reached. The control algorithm is defined by the control law as given in equation (Figure 1).

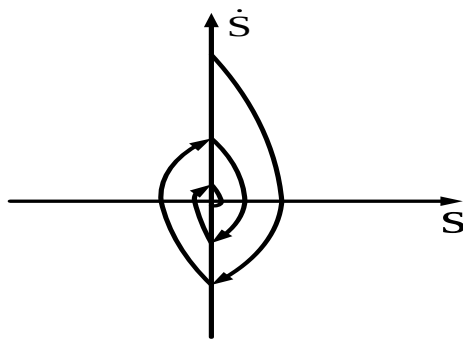


Figure 1. Trajectory of s and $\dot{s}$ dot (Twisting).

$$\dot{u}_{TW} = \begin{cases} -u_{TW} & \text{for } |u| > u_{\max} \\ -k_m \text{sign}(s) & \text{for } s\dot{s} \leq 0 \text{ and } |u| \leq u_{\max} \\ -k_M \text{sign}(s) & \text{for } s\dot{s} > 0 \text{ and } |u| \leq u_{\max} \end{cases}$$

Where

$$k_M > k_m > 0, k_m > \frac{4\Gamma_M}{s_0},$$

$$k_m > \frac{\phi}{\Gamma_m}, \Gamma_m k_M - \phi > \Gamma_M k_m + \phi$$

Super Twisting for Second Order Plus Delay (SOPDT) Model

The trajectories plotted with the s and $\dot{s}$ dot variables are twisted around the origin with travel on the $\dot{s}$ dot axis (super-twisting) using the super twisting method. The super twisting sliding mode control algorithm incorporates an integrator into the control loop, making control a continuous time function (ust). The control law u_{ST} is defined by two terms: the first is an integral of a discontinuous sliding variable function, while the second is a continuous sliding variable function. A mathematical equation is used to define the control law (Figure 2).

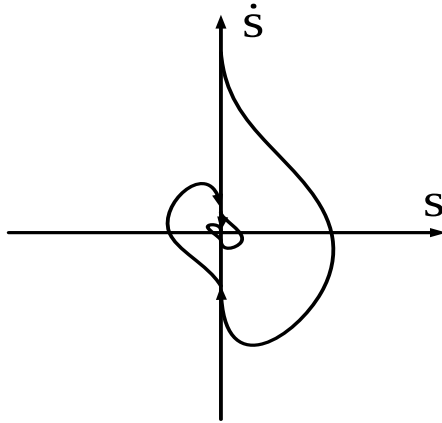


Figure 2. Trajectory of s and $\dot{s}$ (Super twisting).

$$\begin{aligned}
 u_{ST} &= u_1 + u_2 \\
 \dot{u}_1 &= \begin{cases} -u_{ST}, & \text{for } |u_{ST}| > u_{\max} \\ -w \text{sign}(s), & \text{for } |u_{ST}| \leq u_{\max} \end{cases} \\
 u_2 &= \begin{cases} -\lambda |s_0|^\rho \text{sign}(s), & \text{for } |s| > s_0 \\ -\lambda |s|^\rho \text{sign}(s), & \text{for } |s| \leq s_0 \end{cases} \\
 w &> \frac{\phi}{\Gamma_m}, 0 < \rho \leq 0.5
 \end{aligned} \tag{8}$$

Where

$$w > \frac{4\Gamma_M}{s_0}, \rho(\lambda\Gamma_m)^{\frac{1}{\rho}} > (\Gamma_M w + \phi)(2\Gamma_M)^{\frac{1}{\rho}-2} \tag{9}$$

Adaptive Algorithm Second Order Plus Delay (SOPDT) model:

Adaptive sliding mode is a useful control technique that outperforms standard SMC in terms of disturbance rejection and chattering avoidance. The adaptive control law is mathematically defined as

$$\dot{u} = -\theta \text{sign}(p)$$

where,

$$\begin{aligned}
 p &= \dot{s} + |s|^{\frac{1}{2}} \text{sign}(s) \\
 \dot{\theta} &= \begin{cases} \bar{\theta} \|p\| \text{sign}(\|p\| - \varepsilon) & \text{if } \theta > \mu \\ \mu & \text{if } \theta < \mu \end{cases}
 \end{aligned}$$

Where $\bar{\theta}$, μ and ε are positive controller parameters to be chosen. The value of μ has been set to restrict the gain to be positive value. The parameter ε defines the region around the switching surface i.e., indicates closeness to the switching surface. It will effect the variation in switching gain and responsible to achieve ideal sliding mode.

Using above algorithms of second order sliding mode controller for regulating second order delay time models like disturbance rejection parameter variation and the impact of damping results are shown in the next chapter.

SIMULATION RESULTS

Twisting Algorithm without Disturbance Conditions

Simulation studies have been carried out by considering the SOPDT system using second order SMC strategy using a twisting algorithm. Here, the parameters $\xi = 0.7$ and $w_n = 3.8$ are considered for

design of switching surface. Different initial conditions considered for simulation studies are $X_0 = [0.0125, 0.0125, 0, 0]$, $[0.05, 0.05, 0, 0]$, $[0.1, 0.1, 0, 0]$ and $[0.075, 0.075, 0, 0]$ with design constants as $V_m = 0.7$ and $V_M = 1$. The history of the state variables x_1 , x_2 , x_3 and the switching surface with time are shown respectively using (Figures 3–7) considering these initial conditions. It is observed that these state variables are returning to the stable equilibrium state using SMC with twisting control algorithm.

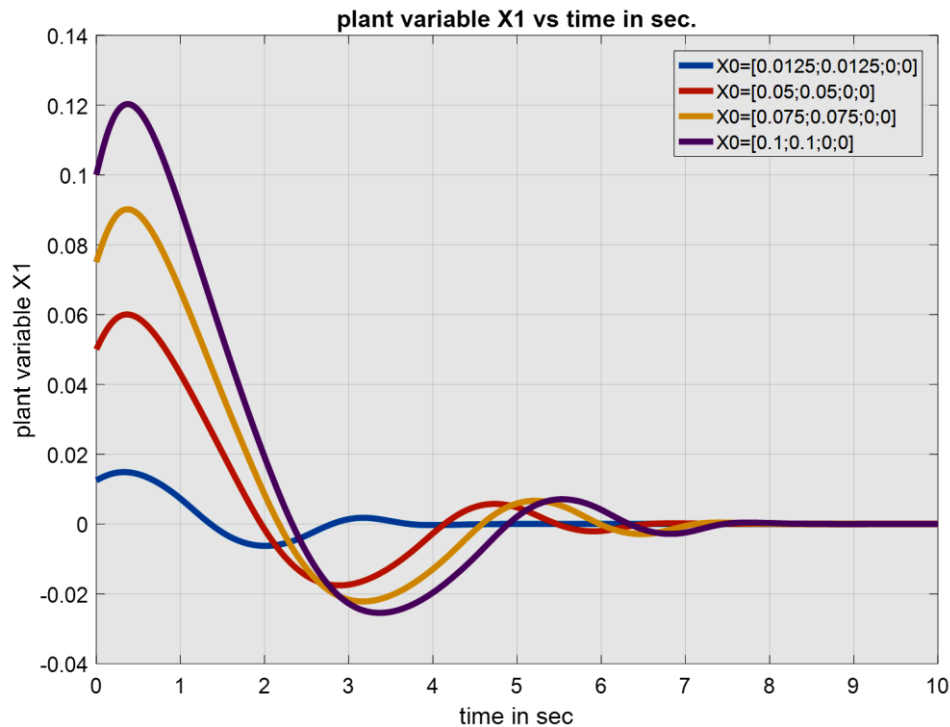


Figure 3. Plant variable X1 vs time without disturbance conditions.

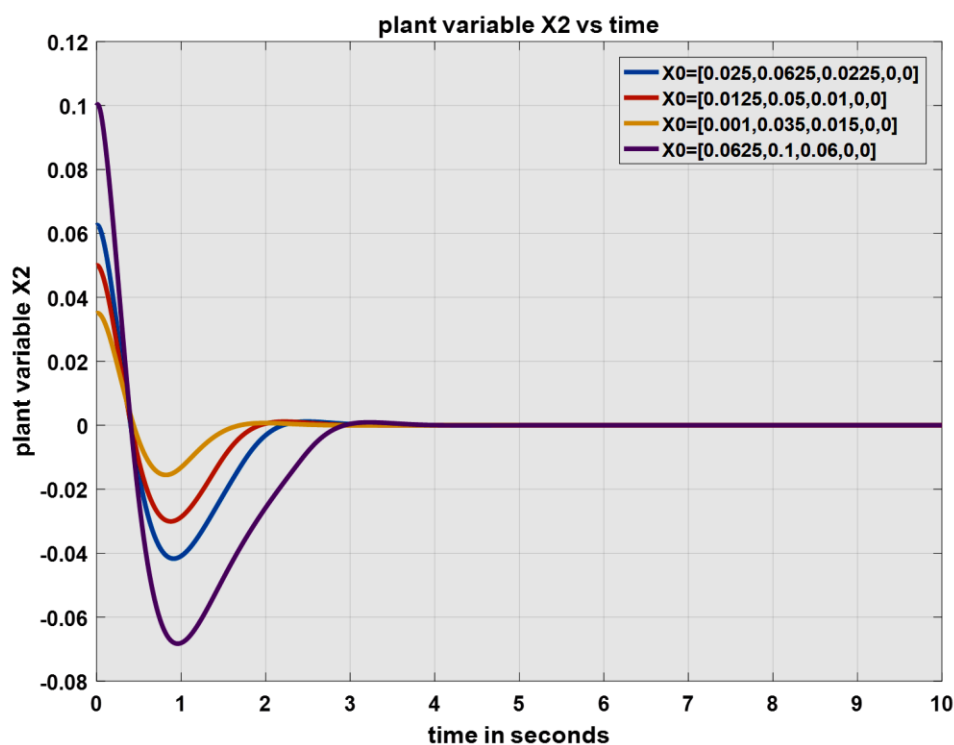


Figure 4. Plant variable X2 vs time without disturbance conditions.

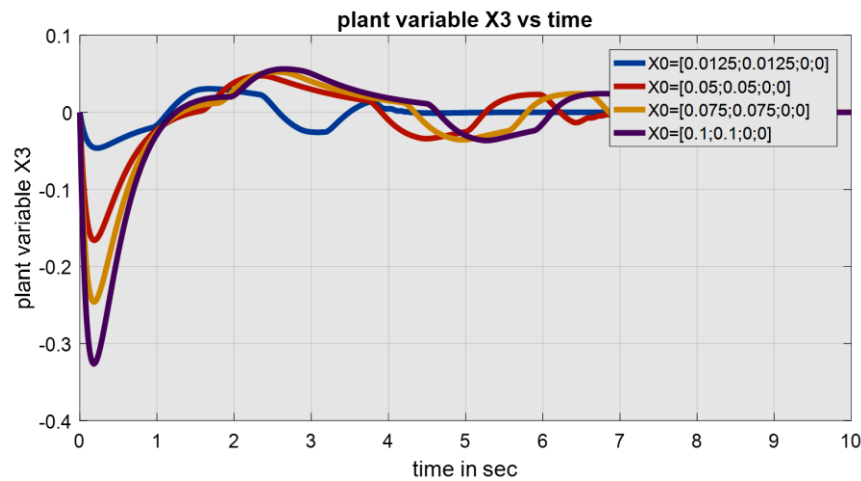


Figure 5. Plant variable X3 vs time without disturbance conditions.

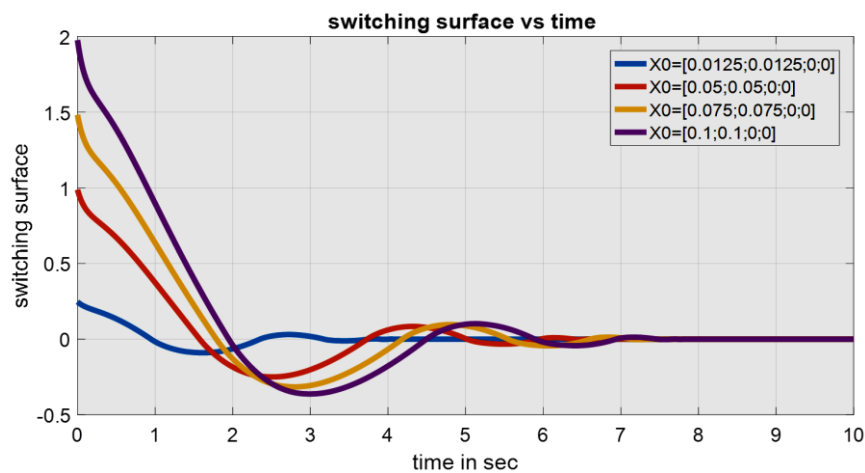


Figure 6. Switching surface vs time under no initial disturbance conditions.

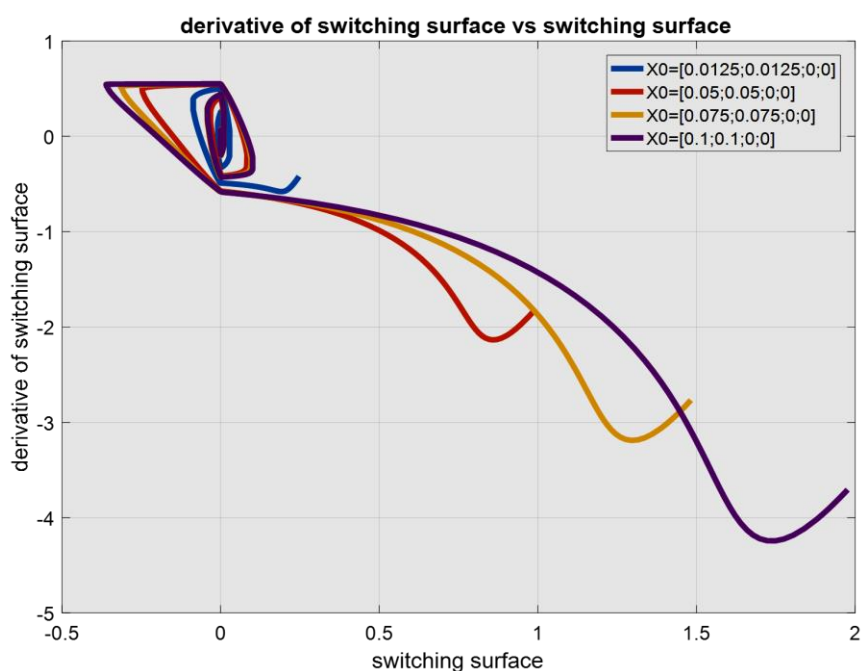


Figure 7. Derivative of switching surface vs switching surface with no disturbance conditions.

The trajectory of switching surface and its derivatives is shown in. The finite time convergence of state trajectories to the origin in the state plane is due to switching of the actual control signal between two different magnitudes. The time taken for the state variables to reach the stable equilibrium point using second order twisting SMC is less compared to first-order SMC.

Twisting Algorithm with Disturbance Conditions

Simulation studies has been carried out by considering the SOPDT system using second order SMC strategy using twisting algorithm with parameter variation are considere for switching surface. $Z = 0.7; 0.5; 0.3$ $\omega_n = 3.8; 2.28; 2.42$; Different initial conditions considered for simulation studies are $X_0 = [0.0125, 0.0125, 0, 0]$, $[0.05, 0.05, 0, 0]$, $[0.1, 0.1, 0, 0]$ and $[0.075, 0.075, 0, 0]$ with design constants as $V_m = 0.7$ and $V_M = 1$. The history of the state variables x_1 , x_2 , x_3 and the switching surface with time are shown respectively using considering these initial conditions. It is observed that these state variables are returning to the stable equilibrium state using SMC with twisting control algorithm (Figures 8, 9).

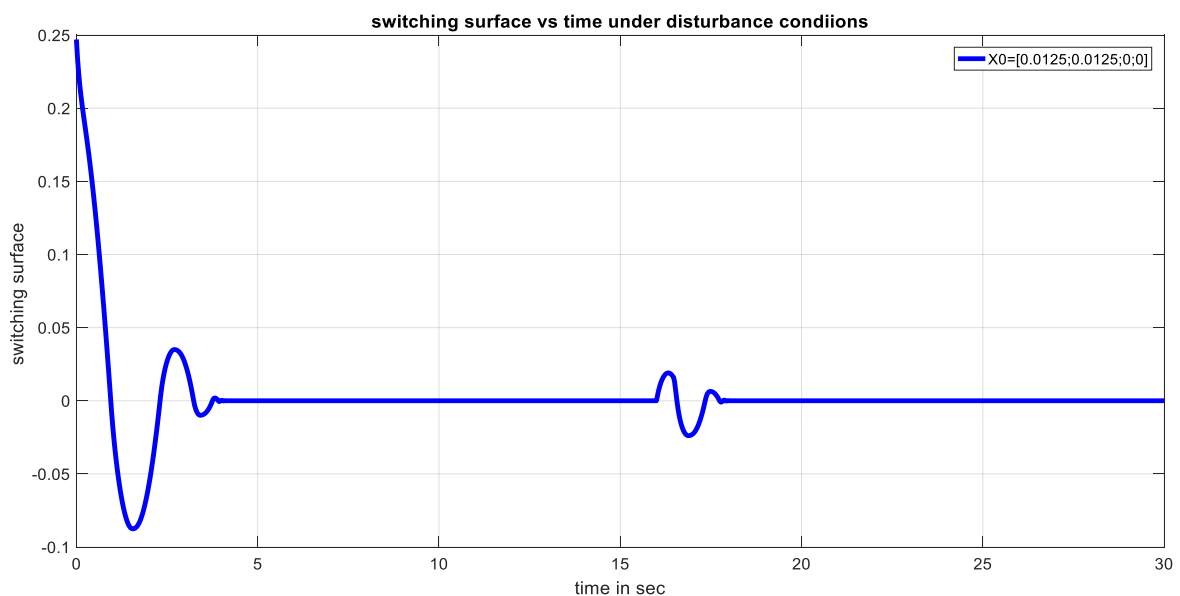


Figure 8. Switching surface vs time under disturbance conditions.

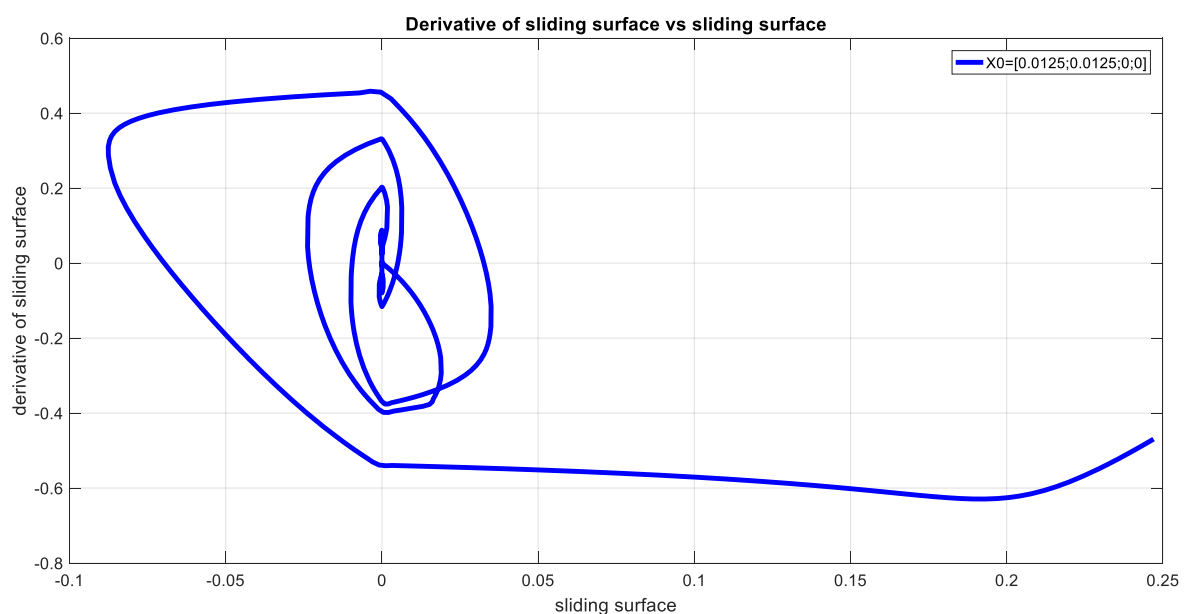


Figure 9. Derivative of switching surface vs switching surface.

For the simulation studies of this system using this algorithm needs important parameters like W , ρ and λ . These are considered as follows $W = 3$, $\rho = 0.5$ and $\lambda = 4.75$. The trajectory of S and S_{dot} using twisting algorithm without disturbance for different initial conditions are graphically shown in Figure 7. It is observed that under different conditions the shape of this trajectory is following desired.

Super Twisting Algorithm without Disturbance Condition

Figure (10–13) derivative of switching surface vs switching surface without disturbance.

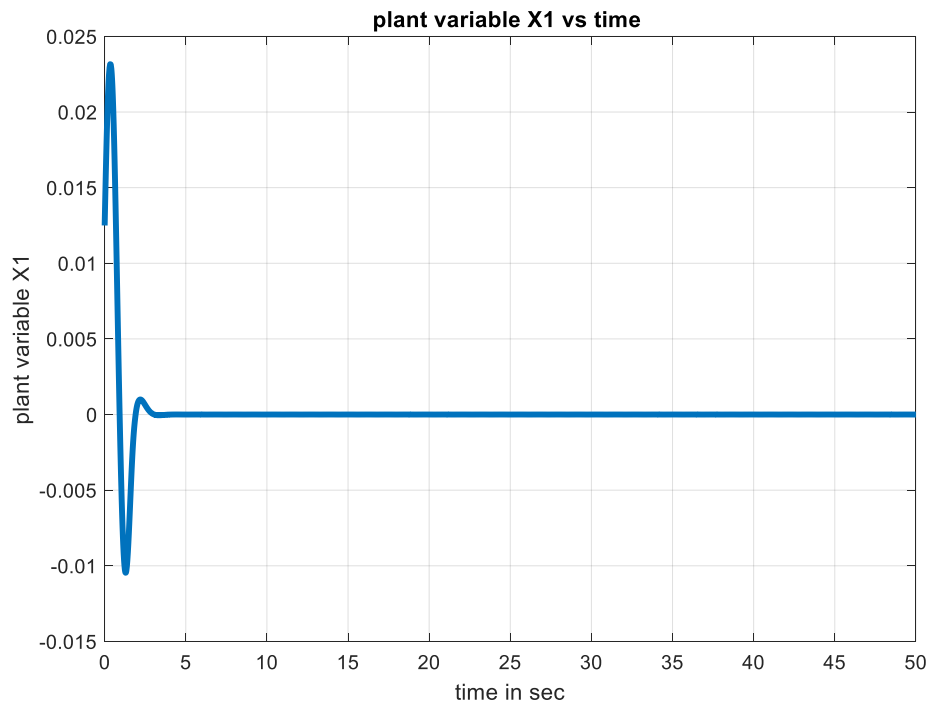


Figure 10. Plant variable X1 vs time without disturbance.

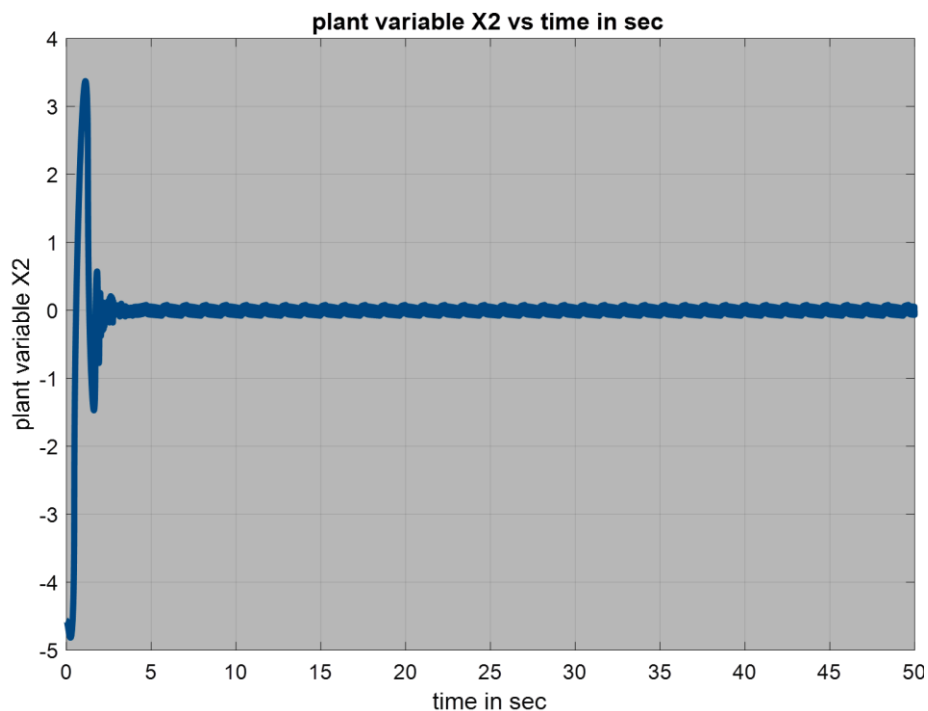


Figure 11. Plant variable X2 vs time in sec without disturbance.

For the simulation studies of this system is also performed using this algorithm under disturbance conditions considering the parameters like W , ρ and λ . The controller parameters are chosen such that, they assure the convergence of the state variables to an equilibrium point in a finite time, when this system is subjected to disturbance under different initial conditions. The constants are chosen as $W = 3$, $\rho = 0.5$, $\lambda = 4.75$, $\xi = 0.7$ and $w_n = 3.8$. The variation of plant variables x_1 , x_2 , x_3 , S and $\dot{S}$ with time are shown respectively in (Figures 10–13) under disturbance condition.

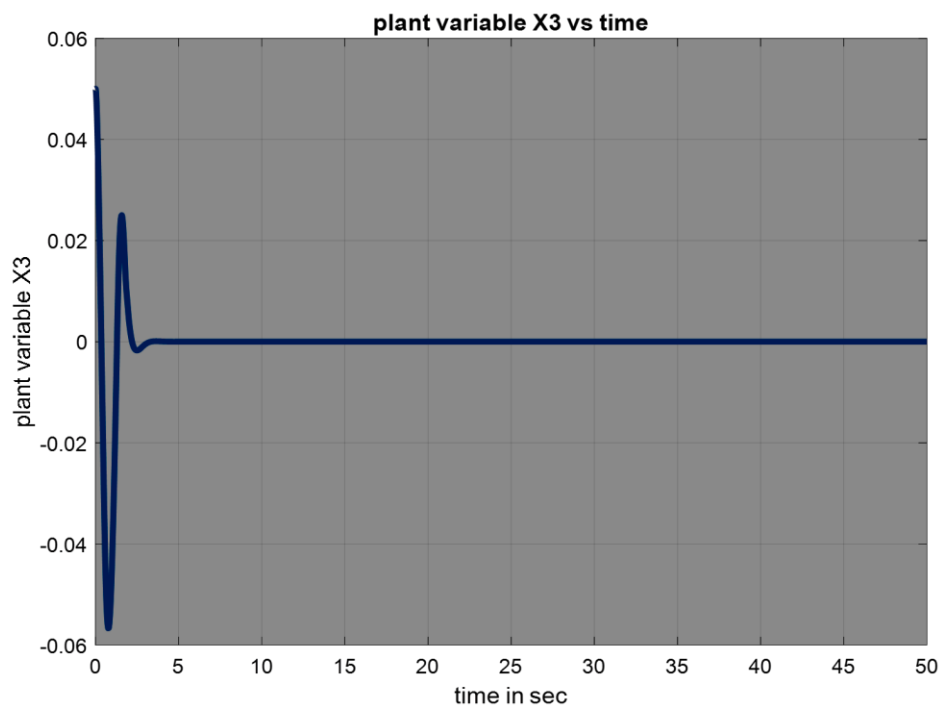


Figure 12. Plant variable X3 vs time under no disturbance condition.

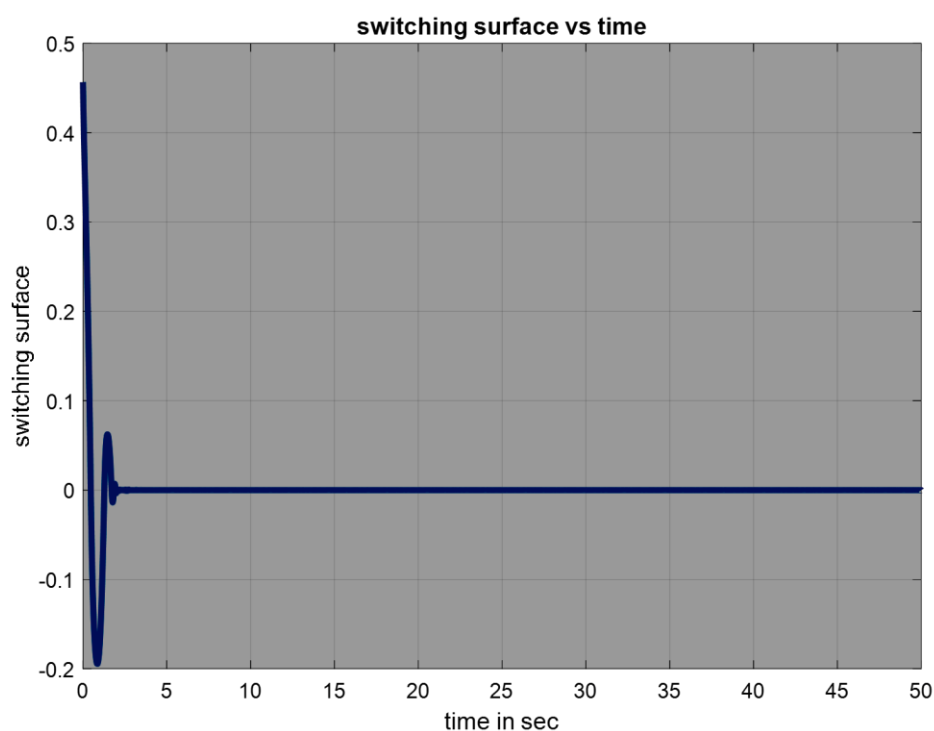


Figure 13. Switching surface vs time under no disturbance condition.

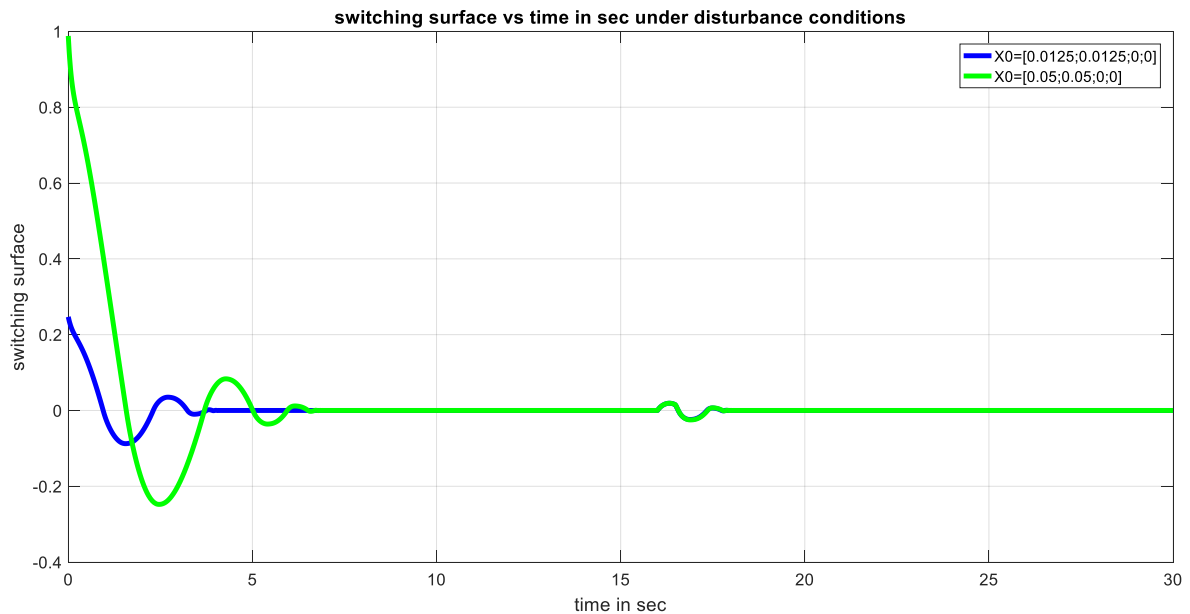


Figure 14. Switching surface vs time in sec under disturbance conditions.

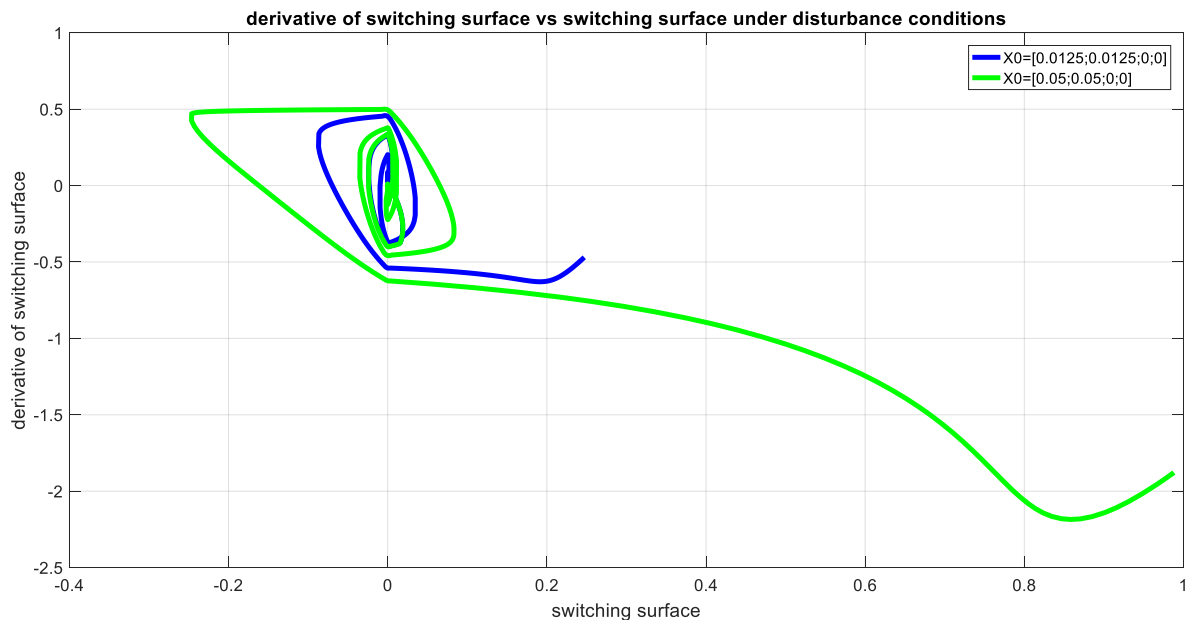


Figure 15. Derivative of switching surface vs switching surface under disturbance conditions.

Super Twisting Algorithm with Disturbance Condition

These are plotted by considering four different initial conditions $x_0 = [0.0125, 0.0125, 0, 0]$, $[0.05, 0.05, 0, 0]$, $[0.075, 0.075, 0, 0]$ and $[0.1, 0.1, 0, 0]$. Here, the system is also subjected to step disturbance $d=0.01*(\text{Heaviside}(t-16)-\text{Heaviside}(t-16.5))$ with a magnitude 0.01 during the period 16 to 16.5 seconds. It is observed that when this system is subjected to the perturbation, these variables are returning to the original equilibrium state. It is observed that the control algorithm is effectively rejecting the disturbances and the regulation characteristics are also good.

Adaptive Algorithm with Parameter Variation

In ASOSMC, $\bar{\Theta}$, μ and ε parameters chosen such that, they assure a finite time convergence of the state variables when this system is subjected to parameter variation. The constants are chosen as, $\bar{\Theta} =$

540, $\mu = 95$ and $\varepsilon = 0.0001$. The system is subjected to parameter variations in damping ratio and natural frequency. The response is plotted with respect to time. The variations of the state variables, sliding surface and derivative of sliding surface with respect to time under the parameter variation conditions are obtained graphically in the interval 0.10. For three different damping ratios 0.3, 0.5, 0.7 and natural frequencies w_n as 2.42, 2.28, 3.8. The parameter variation of plant variables x_1, x_2, x_3, s and $\dot{s}$ with time are shown respectively in Figures 14, 15. It is observed that this SMC is effective under these conditions (Figures 16 to 20).

Adaptive Algorithm with Disturbance

ASOSMC provides smooth control to force the system states on sliding surface. There is no chattering effect and also insensitive to the parameter variation Figures 21–24.

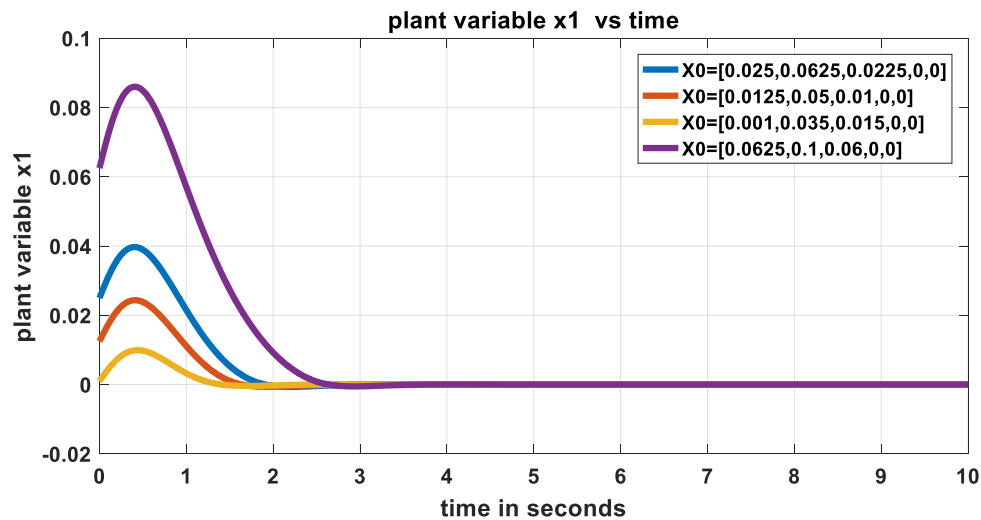


Figure 16. Plant variable X1 vs time under no disturbance conditions.

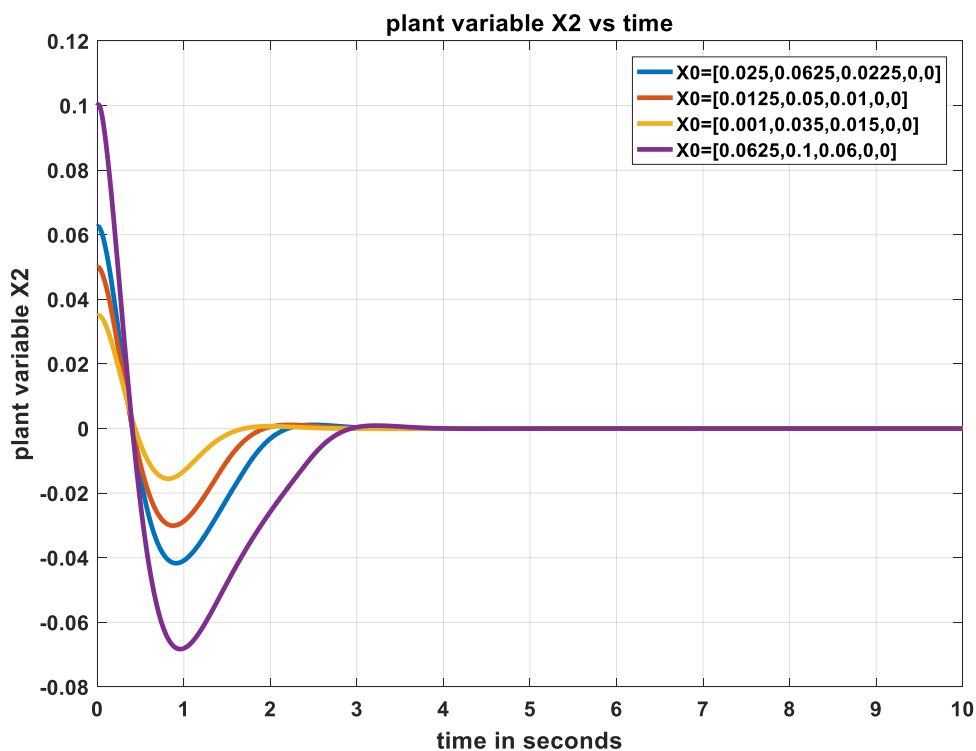


Figure 17. Plant variable X2 vs time under no disturbance condition.

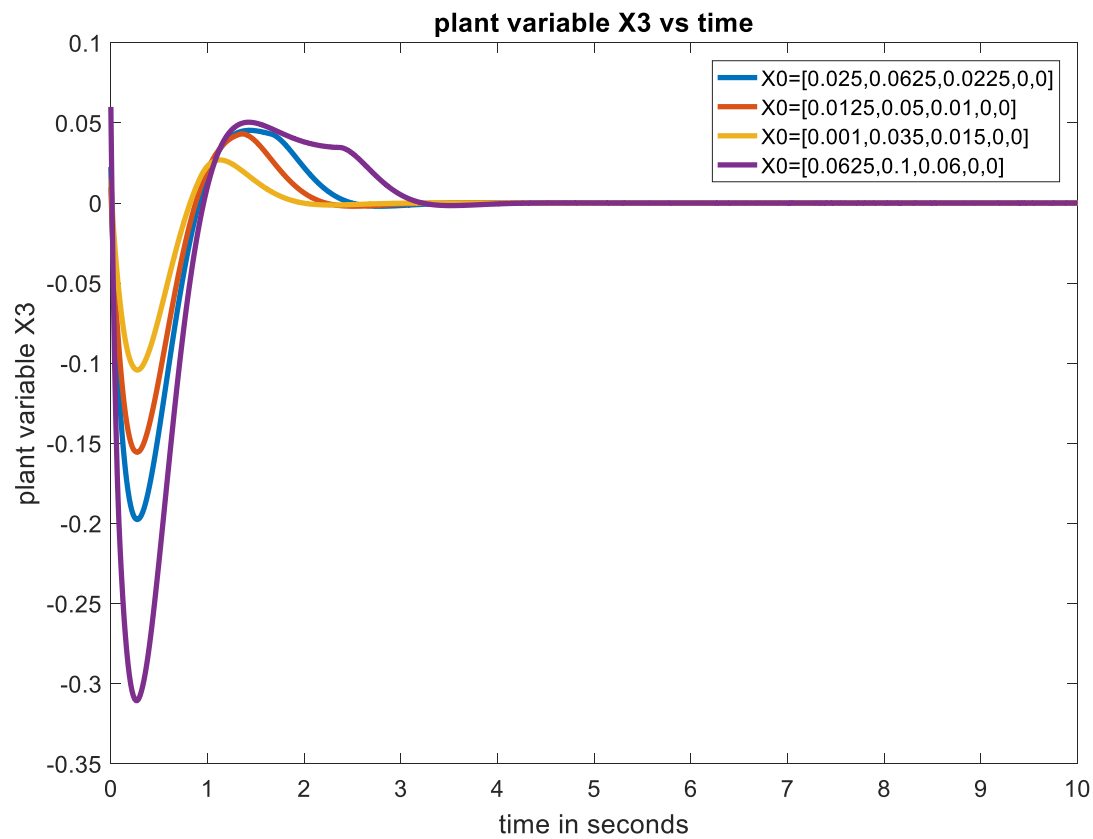


Figure 18. Plant variable X3 vs time under effective disturbance condition.

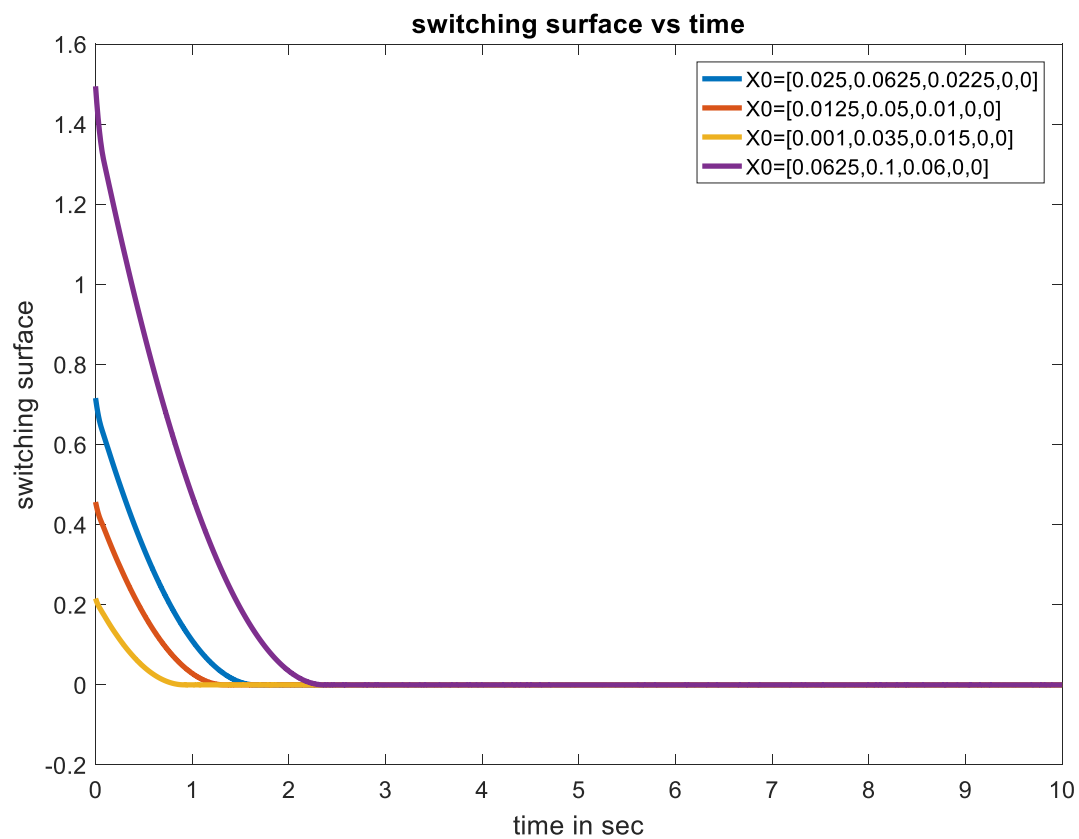


Figure 19. Switching surface vs time under effective disturbance conditions.

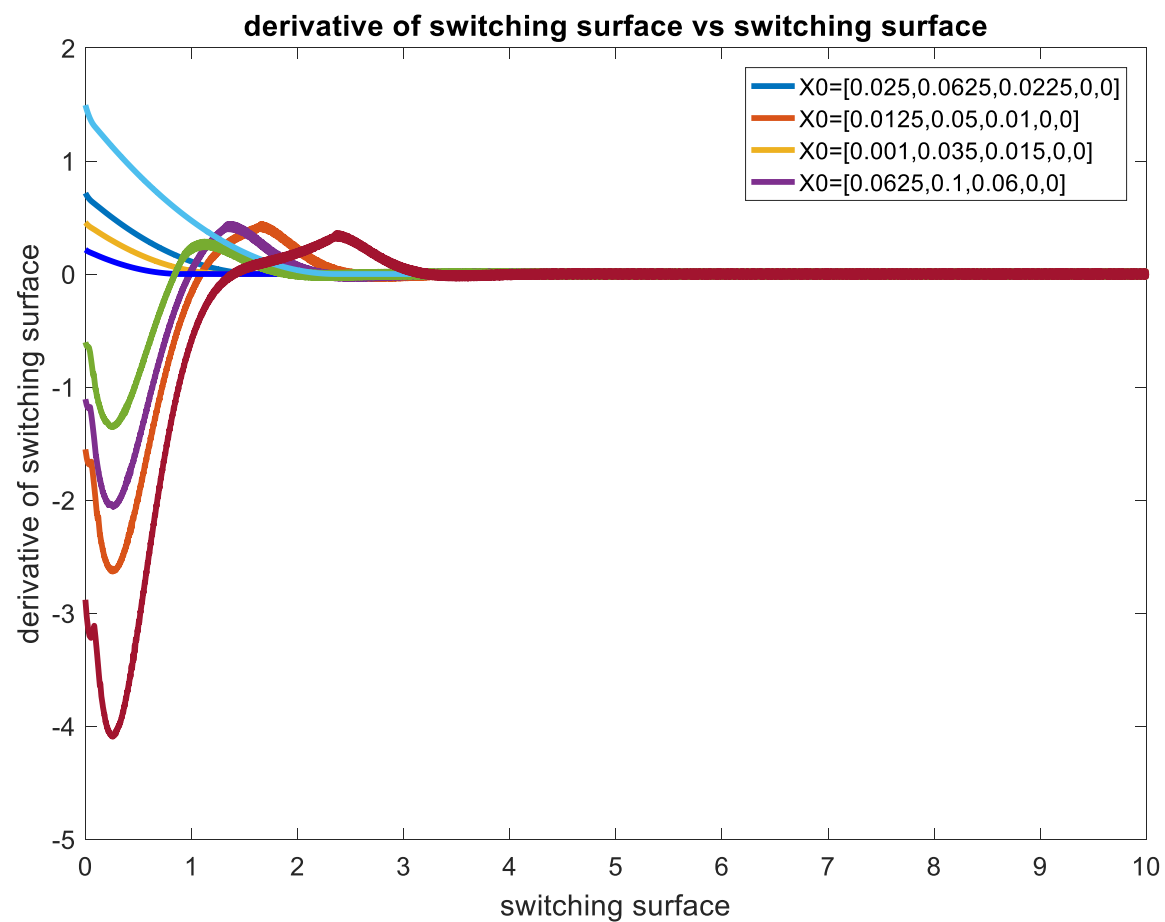


Figure 20. Derivative of switching surface vs switching surface under no disturbance conditions.

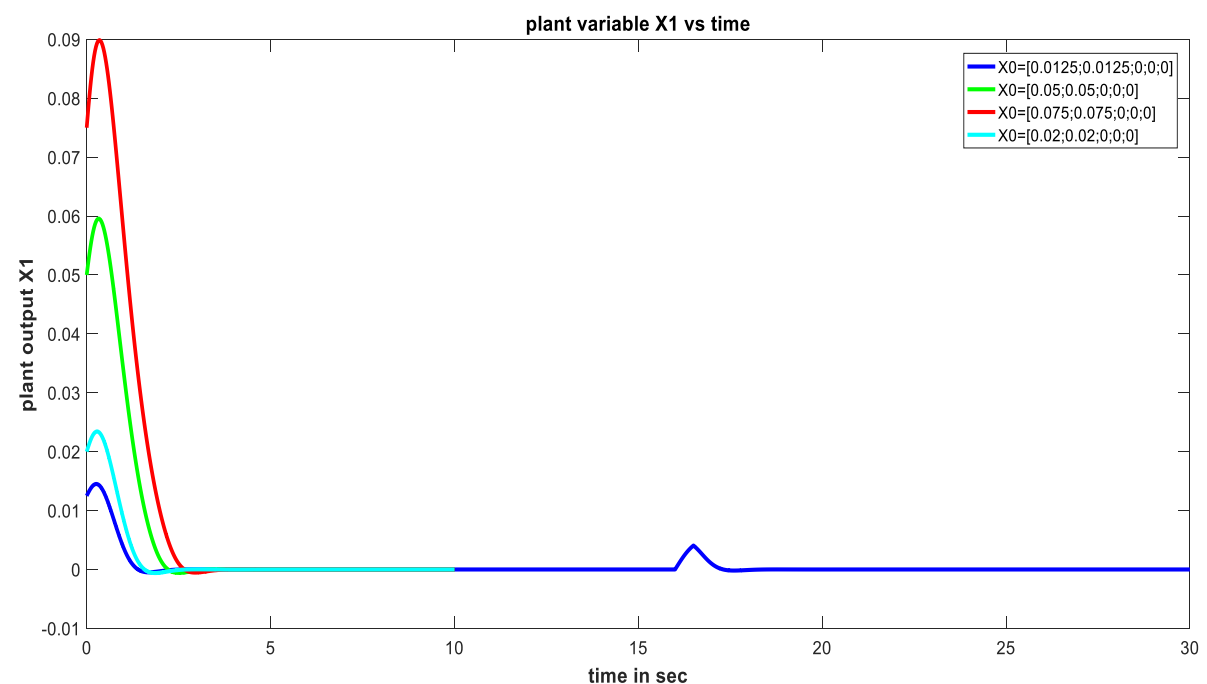


Figure 21. Plant variable X1 vs time under disturbance condition.

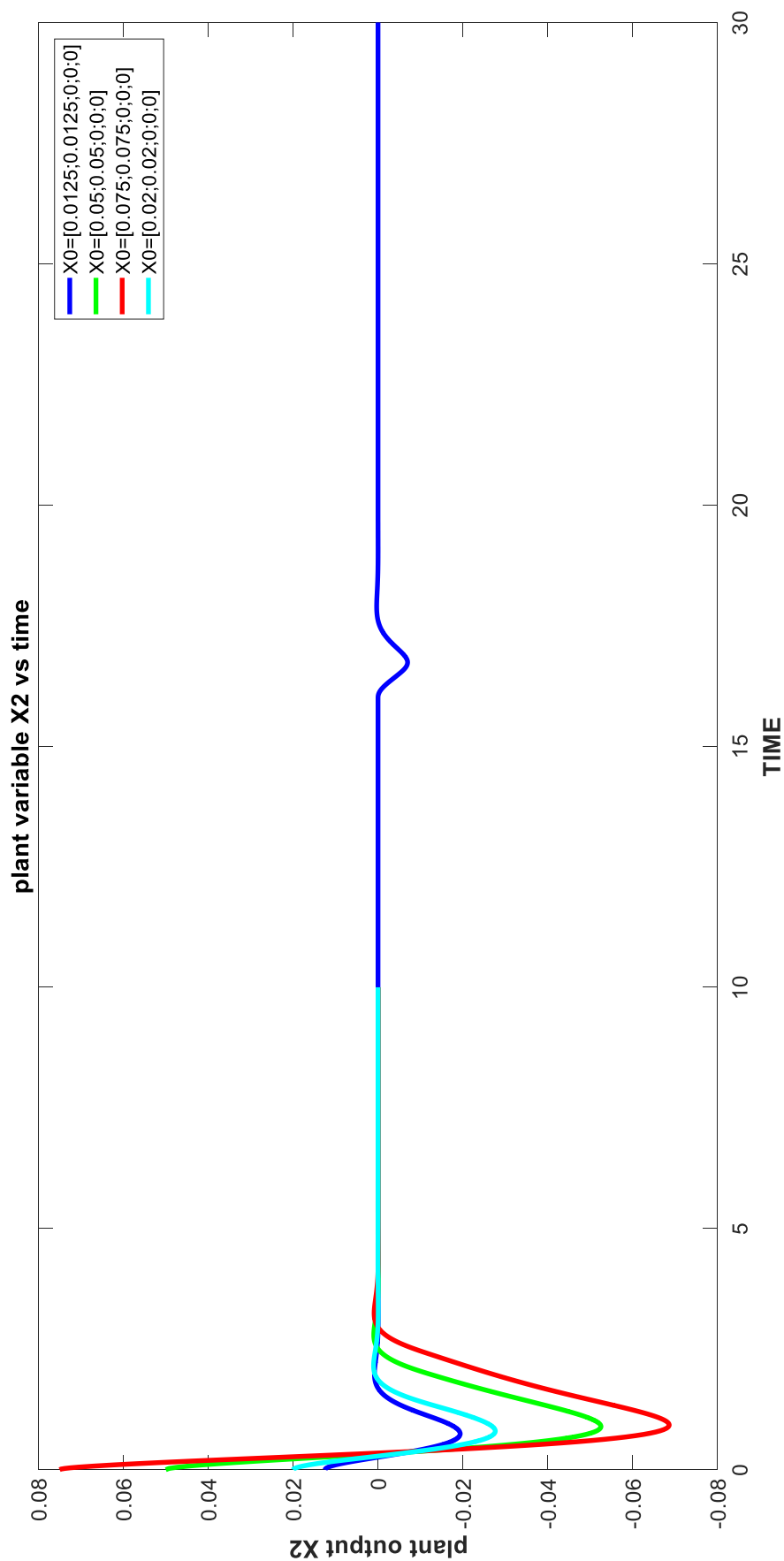


Figure 22. Plant variable X2 vs time under disturbance condition.

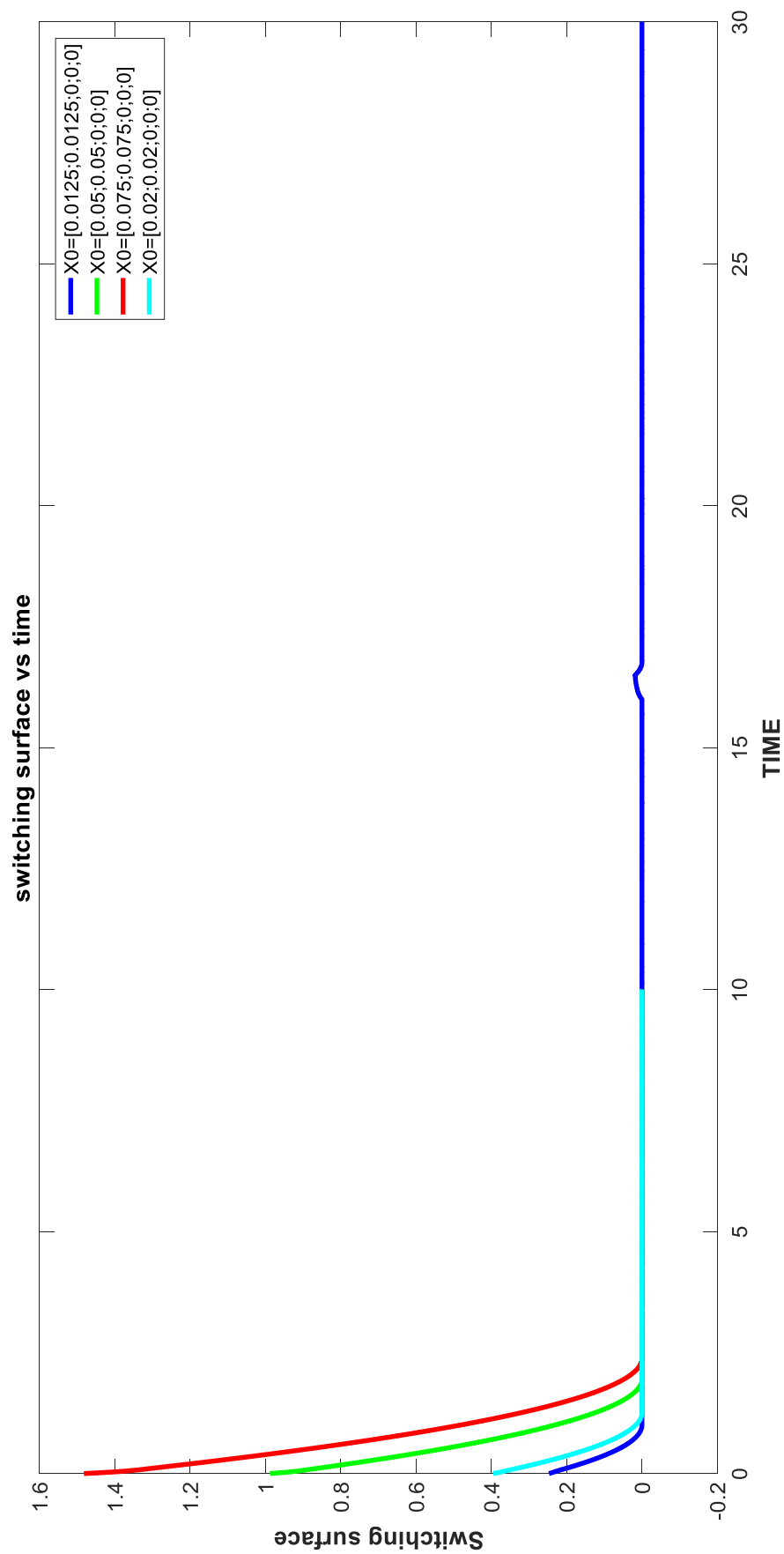


Figure 23. Switching surface vs time.

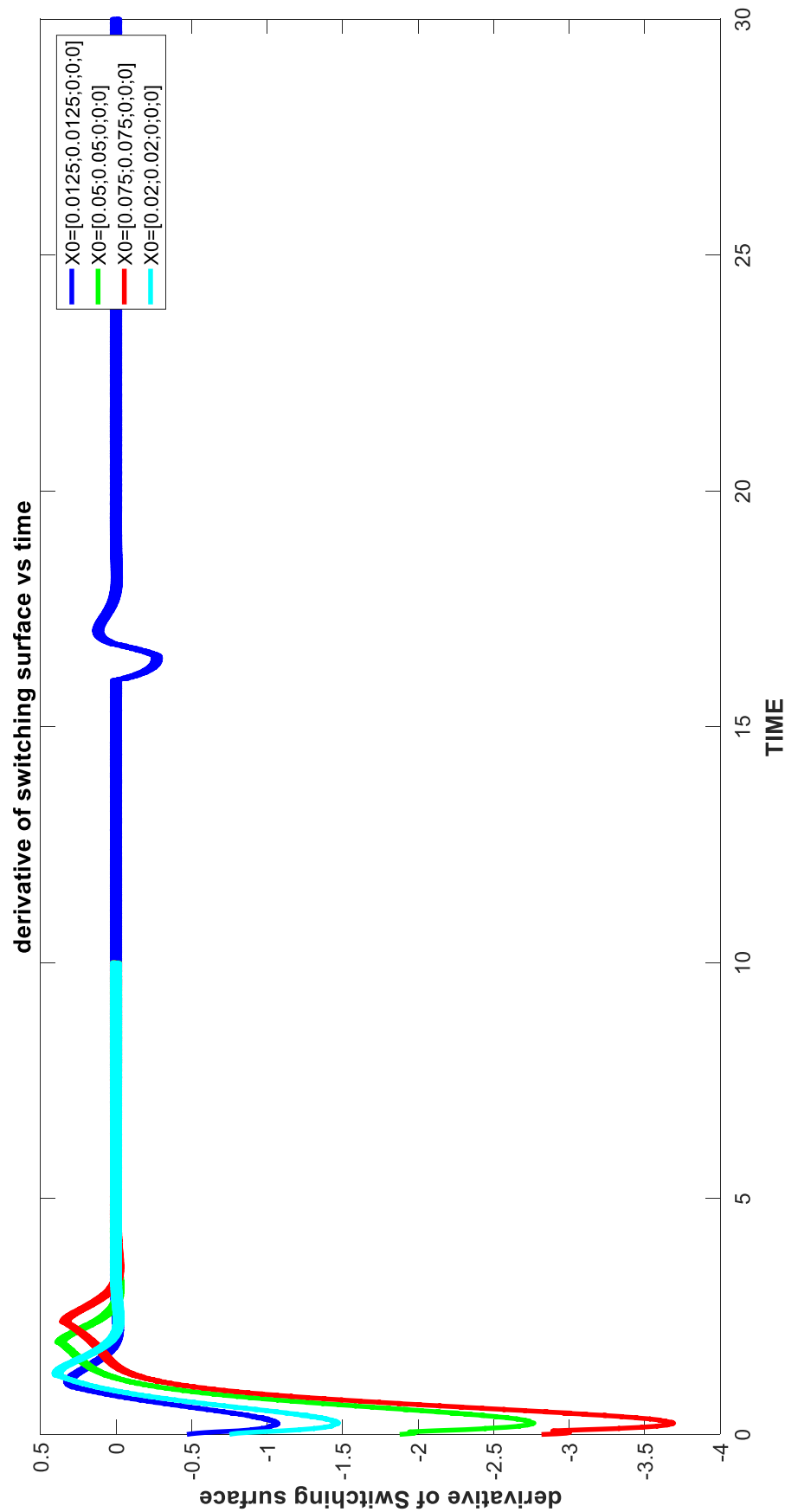


Figure 24. Derivative of switching surface vs time.

CONCLUSIONS

SOSMC was designed using the different sliding surfaces twisting, super twisting, and adaptive algorithms to optimize the performance of the second order processing with a time delay Existence of SOSMC,SOPDT systems with sliding mode have been demonstrated. Significant improvements in results. The suggested low overshoot and small overshoot are possible with these strategies. Settling time at the same moment Also strong and high-quality. The above can be used to regulate performance. SOSMC algorithms are described. We can see from the simulation results that SOSMC provides control while avoiding chattering issues. As a result, the SOSMC algorithms used give an improvement in transient performance as well as steady state accuracy with smooth control efforts and settling time.

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